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An Algorithm for Determining the Location of a Segment on a Segmented Image

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Abstract. The task of digital image analysis after image segmentation requires the ability to operate independently with each extracted segment (object), determine the total number of segments, and define the location of the segments on the image plane. This requires knowledge of the coordinates of each extracted segment on the image. This involves defining the coordinates of the pixels that make up the segment.

For this purpose, an original algorithm was developed, which during the process of implementation of early developed non-parametric segmentation algorithm extracts the connected components (segments) on the image and determines the location of the each segment on the image based on the values of indices i and j (coordinates) of its pixels.

Key words: cluster domain; cluster; density function; matrix cluster; mode; non-parametric segmentation; Parzen estimation; segment.

Introduction

The task of digital image analysis after image segmentation requires the ability to operate independently with each extracted segment (object), determine the total number of segments, and define the location of the segments on the image plane. This requires knowledge of the coordinates of each extracted segment on the image. This involves defining the coordinates of the pixels that make up the segment.

For this purpose, an original algorithm was developed, which during the process of implementation of early developed non-parametric segmentation algorithm extracts the connected components (segments) on the image and determines the location of the each segment on the image based on the values of indices i and j (coordinates) of its pixels.

Main part

Segmentation Algorithm: Brief Overview

Here we'll briefly introduce the early developed segmentation technique [1]. That involves two stages. The first is cluster-analysis of the k-dimensional characteristic features space, in which number of clusters, their centres and radiuses of sameness are determined; and the second is the extraction of segments on the digital image on the bases of mapping the obtained clusters back to the image domain.

Scene digital image is considered as a realisation of a vector random field

$$G = f(x,y) = \left(f^{(1)}(x,y),\dots,f^{(k)}(x,y)\right),$$

with independent components and increases, where in each image point (x,y) we have row vector G of k characteristic features i.e. as a matrix

$$A = \|a_{ij}\|, i=\overline{1,N}, j=\overline{1,M},$$

where each a_{ij} is an observed value of a vector random field in 2D space point (x_i,y_j) and

$$a_{ij} = \left(a_{ij}^{(1)},\dots,a_{ij}^{(k)}\right),$$

where $x_1 < x_2 < \dots < x_N, y_1 < y_2 < \dots < y_M$.

Let's consider in matrix A so-called 2D "window" of size (2×2), in which is defined 6 pairs of adjacent elements. Move the "window" in A element by element and calculate the Euclidean distances $\rho(\cdot)$ between those pairs of adjacent elements.

$$\rho(a_{ij};a_{i,j+1}), \rho(a_{ij};a_{i+1,j+1}), \rho(a_{ij};a_{i+1,j}),$$
$$\rho(a_{i+1,j};a_{i,j+1}), \rho(a_{i+1,j};a_{i+1,j+1}),$$
$$\rho(a_{i,j+1};a_{i+1,j+1}), i=\overline{1,N-1}, j=\overline{1,M-1}. \quad (1)$$

We'll obtain sequence of non-negative numbers $\{\xi_q\}, q=\overline{1,n}$, where $n = 4(N-1)(M-1)+N+M-2$ and ξ_q denotes qth member of sequence $\{\rho(\cdot\,;\cdot)\}$, and which is considered as a sample of a parent population with theoretical density function $\varphi(x)$. For statistical estimation of unknown $\varphi(x)$ the Parzen function is considered

$$\widehat{\varphi_n}(x) = \frac{1}{n\cdot h}\sum_{q=1}^{n} K\left(\frac{x-\xi_q}{h}\right), \quad (2)$$

where $h = h(n) > 0$ and $h(n)\to 0$, $nh(n)\to\infty$, when $n\to\infty$; $k(x)$ is a Borel function which can be integrated relative to Lebesgue measure.

If one takes into consideration that $\varphi(x)$ uniformly continuous on whole numerical axis, then $\widehat{\varphi_n}(x)$ converges to $\varphi(x)$ uniformly with probability 1 and consequently computations are true with probability 1. For definite types of kernels $k(x)$ function $\widehat{\varphi_n}(x)$ is characterized with multi-modes. If $M_i, i=\overline{1,p}$ are modes and m_i are points of local minima of density function $\widehat{\varphi_n}(x)$, then we have $m_i \le M_i < m_{i+1}$. Hence modes are considered as centers of extracting in the sequence $\{\xi_q\}, q=\overline{1,n}$ clusters, while intervals $[m_i;m_{i+1}), i=\overline{1,p}$ formulate so-called clusters domain which give us rough boundary of clusters. But only some of the elements of sequence $\{\xi_q\}, q=\overline{1,n}$ fall into ith interval $[m_i;m_{i+1})$. Let's say $\xi_{i_1},\xi_{i_2},\dots,\xi_{i_e}\in\{\xi_q\}$. The differences $\left|\xi_{i_j}-M_i\right|, j=\overline{1,e}$ give us value of distances of those elements from the cluster center M_i. Let's consider them as a sample of a parent population, which has some theoretical density function. The Parzen function is computed to that again and maximal of its modes is considered as a radius of sameness R_i for cluster domain $[m_i;m_{i+1})$. R_i give us exact boundary of cluster extracting in cluster domain $[m_i;m_{i+1})$.

All $\xi_q\in[m_i;m_{i+1}), q=\overline{1,n}$ which satisfy the clustering criteria

$$|\xi_q - M_i| \le R_i, i=\overline{1,p}, q=\overline{1,n} \quad (3)$$

define cluster K_i with center M_i and radius R_i.

Segmentation of digital image i.e. matrix A implies its partition into maximal connected disjoint unities of the a_{ij} elements according to each $K_i, i=\overline{1,p}$ cluster. At the same time it should be realized in direction as modes are ordered - $M_1 < M_2 < \dots < M_p$. Thus segments extraction process must start from the clusters with the most similar elements and continues doing the same with all other clusters until we reach the last one that has the least similar elements, i.e. from cluster K_i with a minimum mode M_1 and finish at cluster K_p with a maximum mode M_p. For that, let's move in matrix A

"window" in the same way as before and find pairs of adjacent elements distances (1) between which belong to cluster K_1. As a result we'll obtain a set of elements called "matrix" cluster A_1 in matrix A. A maximal connected unities of elements in A_1 create segments $S_1^t, t = \overline{1, T_1}$. If after that in matrix A will remain non-segmented elements, we pass to cluster K_2 and repeat above-mentioned procedure, etc. until process of segmentation of the matrix A will not be terminated. At the same time the matrix A elements extracted into segments on ith step do not take part in extraction of segments on $(i + 1)$ step. The elements of image extracted into segments on ith step are mark with one and the same identifying label whose value is correspond to cluster's current number. After the termination of segmentation procedure some elements of matrix A can be remained as non-segmented and distributed on the image domain. Such elements represent isolated elements on the image plane.

Algorithm Description

According to the above-mentioned segmentation algorithm, at the ith step of the segmentation procedure the elements of the A_i matrix cluster, corresponding to cluster K_i, $i = \overline{1, p}$ are assigned the same labels. The value of the remaining elements of the matrix is equal to 0. Thus, we get a binary matrix cluster with labeled elements. The task is to extract in such a matrix cluster the connected components - segments on the image on the bases of the elements marked with the labels corresponding to the cluster K_i, $i = \overline{1, p}$ and define the coordinates of each of them based on the values of the i and j indices of their pixels. This will ultimately determine the location of the segment on the image.

Suppose we have corresponding to cluster K_i, $i = \overline{1, p}$ matrix cluster A_i, in which the elements are labeled with a value corresponding to ith cluster, say, equal to 5, and the remaining elements of the matrix are equal to 0 (Figure 1).

The task is to define the corresponding segment and its location on the image plane based on the extraction of the maximal subset of the connected elements in the A_i matrix cluster. The latter involves determining the coordinates of the pixels that make up the obtained segments. To do this, start the element by element movement in the matrix cluster A_i (left to right and top to down) and compose the corresponding sequences of the indices i and j of elements with a value of equal to 5 in the matrix. The result is the following sequences of indices i and j:

$$i = 2,2,3,3,3,3,3,3,4,4,4,7,7,7,7$$
$$j = 2,3,1,2,3,4,5,6,4,5,6,3,4,5,6$$

If we now sort these sequences in ascending order without repeating the same index values, then we get the following two sequences:

$$i = 2,3,4,7$$
$$j = 1,2,3,4,5,6$$

As we can see, in the case of index i, starting from the value $i = 2$ and including $i = 4$, the index runs through all natural values, that is, the difference between two adjacent values in the sequence is 1. Accordingly, the matrix cluster has labeled elements under these index values i. Then we no longer have the values of the index i from the value $i = 4$ to the value $= 7$. This means that there are no labeled elements in the fifth and sixth rows of the matrix cluster. Labeled elements start from the seventh row. Therefore, the set of elements labeled by rows in the matrix A_i is divided into two connected subsets. Whereas index j runs all natural values from 1 to 6, i.e. labeled elements form a single connected set by columns. Therefore, the sequence of the values of index i is divided into 2 subsequences: 2,3,4 and 7, while the sequence of the values of index j remains unchanged.

0	0	0	0	0	0	0	0	0	0
0	5	5	0	0	0	0	0	0	0
5	5	5	5	5	5	0	0	0	0
0	0	0	5	5	5	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	5	5	5	5	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Figure 1

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Thus, in order to extract the maximal number of connected elements in the matrix cluster A_i, we need to consider the following combinations of the values of the sequences of indices i and j:

$$i = 2,3,4$$
$$j = 1,2,3,4,5,6 \quad (4)$$

and

$$i = 7$$
$$j = 1,2,3,4,5,6 \quad (5)$$

The above means that the set of labeled elements of the matrix A_i, corresponding to the cluster K_i is divided into 2 connected subsets. One of these is placed inside the area (interval) bounded by the values of indices i and j

$$i = [2,3,4]$$
$$j = [1,2,3,4,5,6] \quad (6)$$

defined by sequence (4). To determine the exact indices of the elements of a connected subset in area (6), we check whether all possible combinations of the pairs of indices i and j correspond to the labeled elements in the matrix cluster A_i. To do this, we will use the following procedure: for all possible combinations of the pairs $[i, j]$ of indices i and j taken from (6), we check the values of the corresponding elements labels in the matrix cluster A_i. If it is equal to 5, then this is the element of the first connected subset. After going through this procedure, we will get the exact values for indices i and j of the elements for the first connected subset in the matrix cluster A_i. So, a segment on the image corresponding to the first connected subset is formed by pixels with the following coordinates:

[2,2], [2,3], [3,1], [3,2], [3,3], [3,4], [3,5], [3,6], [4,4], [4,5], [4,6].

According to (5), the second connected subset in the matrix cluster A_i is placed inside the area (interval) bounded by the values of indices i and j:

$$i = [7]$$
$$j = [1,2,3,4,5,6] \quad (7)$$

If we now repeat the above-mentioned procedure again for all possible combinations of the pairs $[i, j]$ of indices i and j taken from (7), we will get the exact values for indices of the elements for the second connected subset in the matrix cluster A_i. Accordingly, a segment on the image corresponding to the second connected subset is formed by pixels with the following coordinates:

[7,3], [7,4], [7,5], [7,6].

It follows, that if indices i and j of the labeled elements of a matrix cluster sequentially run natural values, then their corresponding pixels on the image form single maximal connected set - a connected component or segment.

Now, consider the case when the corresponding to cluster K_i, $i = \overline{1, p}$ matrix cluster A_i has the form shown on Figure 2.

0	0	0	0	0	0	0	0	0	0
0	5	5	0	0	0	0	0	0	0
0	0	5	5	0	0	0	0	0	0
0	0	0	5	5	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	5	5	5
0	0	0	0	0	0	0	5	5	5
0	0	0	0	0	0	0	0	0	0

Figure 2

Let's repeat the above-mentioned procedure again for the given matrix cluster and compose the sequences from the values of indices i and j of the labeled elements. We'll obtain:

$$i = 2,3,4,6,7$$
$$j = 2,3,4,5,8,9,10 \quad (8)$$

As we can see, both indices i and j do not run sequentially the natural values. In particular, the interrupt at index i occurs from the fourth to the sixth row, i.e. there are no labeled elements in the fifth row. Labeled elements start from the sixth row. This also happens in the case of index j. Here also the interrupt occurs from the fifth to the eighth column, i.e. there are no labeled elements in the sixth and seventh columns. Labeled elements start from the eighth column. Thus, respectively, each sequence of values of indices i and j is divided into two subsequences, respectively:

2,3,4 and 6,7;

and

2,3,4,5 and 8,9,10.

Therefore, to extract the set of maximal number of connected elements in matrix cluster, we must consider the following combinations of the subsequences of values of the indices i and j:

$$i = 2,3,4 \quad j = 2,3,4,5, \qquad (9)$$

$$i = 2,3,4 \quad j = 8,9,10 \qquad (10)$$

$$i = 6,7 \quad j = 2,3,4,5, \qquad (11)$$

$$i = 6,7 \quad j = 8,9,10 \qquad (12)$$

Thus, the set of labeled elements in the matrix cluster A_i is divided into two connected subsets, respectively, both by the indices i and j. Thus, we can consider 4 areas of possible existence of segments on the image. This means to determine if there are subsets of connected elements within the areas bounded by the values of these indices in the matrix cluster. Let's consider these cases for each area separately:

1.

$$i = [2,3,4] \quad j = [2,3,4,5]$$

To define a connected subset and its exact coordinates in the area defined by (9), we first check whether all possible combinations of the pairs $[i, j]$ of indices i and j taken from (9) correspond to the elements labeled in the matrix cluster A_i. For this we use the following procedure: for all possible combinations of the pairs $[i, j]$ of indices i and j taken from (9), we check the value of the label of the corresponding elements in the matrix cluster A_i. If it is equal to 5, then it is an element of the first connected subset. After going through this procedure for all pairs of indices, we will get the exact indices i and j for the elements of the first connected subset in the matrix cluster A_i. Thus, a segment corresponding to the first connected subset in the matrix cluster is formed on the image by pixels with the following coordinates:

[2,2], [2,3], [3,3], [3,4], [4,4], [4,5]

2.

$$i = [2,3,4] \quad j = [8,9,10]$$

We repeat the same procedure in the matrix cluster A_i for the area bounded by the values of the indices (10). None of the all possible combinations of the pairs $[i, j]$ of indices i and j correspond to the elements labeled in the matrix cluster A_i. Thus, the elements of the matrix cluster defined by these indices values do not form a segment on the image.

3.

$$i = [6,7], \quad j = [2,3,4,5]$$

Here we have the same case for the area bounded by the values of indices (11) as in case (10).

4.

$$i = [6,7] \quad j = [8,9,10]$$

For all possible combinations of the pairs $[i, j]$ of indices i and j taken from (12), we repeat the procedure described in case 1. We see, that to all possible combinations of the pairs $[i, j]$ of indices i and j in the matrix cluster A_i correspond the elements labeled equal to 5. As a result, a connected subset of the elements of the matrix cluster with these indices corresponds to a segment in the image whose pixel coordinates are:

[6,8], [6,9], [6,10], [7,8], [7,9], [7,10]

Computational Algorithm

Based on the above-mentioned, we can formulate the following computational algorithm for determining the location of a segment on a segmented binary image:

Input: corresponding to cluster K_i, $i = \overline{1,p}$ matrix cluster A_i of $n \times m$ dimensionality;

Output: $[i, j]$ coordinates of the pixels that make up the segment on the image;

- Move in the matrix cluster A_i element by element (left to right and top to down) and form two sequences i and j from the indices i and j of the labeled elements corresponding to cluster K_i, $i = \overline{1,p}$;
- Arrange the sequences i and j in ascending order without repeating the same index values;
- We are looking for interrupts in each sequence, i.e. in the places where the difference between two adjacent sequence values is greater than one;
- If there is no interruption in any sequence of indices, then the coordinates of the labeled elements of the matrix cluster, corresponding to these indices form a segment on the image;
- If one or more interrupts occur in one or both of the indices sequences, then the subsequences of indices i and j are formed based on the values of the indices located between the interrupts;
- All combinations of the obtained subsequences are considered, and for each of them the presence of labeled elements in the matrix cluster is checked;
- In the presence of the labeled elements, such ones form the maximal connected subset. Otherwise there is a transition to the following combination of subsequences;
- To a connected subset in the matrix cluster corresponds a segment on the image.

Conclusion

The presented algorithm allows in the process of nonparametric image segmentation to extract the maximal sets of connected pixels - segments on the image, to define the coordinates of each of their elements and, as a consequence, the location of the segments on the image. This allows operating independently with each segment of the image.

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სეგმენტირებულ გამოსახულებაზე სეგმენტის მდებარეობის განსაზღვრის ალგორითმი

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ანოტაცია. ციფრული გამოსახულების ანალიზის ამოცანა გამოსახულების სეგმენტაციის შემდეგ მოითხოვს თითოეულ გამოყოფილ სეგმენტთან (ობიექტთან) დამოუკიდებლად ოპერირების შესაძლებლობას, სეგმენტების საერთო რაოდენობის დადგენას და მათი მდებარეობის განსაზღვრას გამოსახულებაზე. ეს მოითხოვს გამოსახულებაზე გამოყოფილი თითოეული სეგმენტის კოორდინატების ცოდნას. ამ მიზნით შემუშავებულია ციფრული გამოსახულების არაპარამეტრული სეგმენტაციის პროცესში პიქსელების მაქსიმალური ბმული სიმრავლის – სეგმენტის გამოყოფის, მისი თითოეული ელემენტის კოორდინატების და შესაბამისად გამოსახულებაზე მთლიანი სეგმენტის მდებარეობის განსაზღვრის ორიგინალური ალგორითმი. ნაჩვენებია მისი მოქმედების შედეგები მაგალითების გამოყენებით.

საკვანძო სიტყვები: არაპარამეტრული სეგმენტაცია; კლასტერი; მატრიცული კლასტერი; მოდა; პარზენის შეფასება; სეგმენტი; სეგმენტის არე; სიმკვრივის ფუნქცია.

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Алгоритм определения положения сегмента на сегментированном изображении

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Аннотация. Задача анализа цифрового изображения после сегментации требует возможности оперировать с каждым выделенным сегментом (объектом) независимо, определить общее число сегментов и положение каждого сегмента на плоскости изображения. Соответственно, это требует знания координат каждого выделенного сегмента на изображении. Это включает определение координат пикселей, составляющих сегмент. Рассматривается оригинальный алгоритм определения максимального связного множества пикселей - сегмента в процессе непараметрической сегментации цифрового изображения, координат каждого из его элементов и, как следствие, положения сегмента на изображении. Показаны результаты работы алгоритма на примерах.

Ключевые слова: кластер; матричный кластер; мода; непараметрическая сегментация; область сегмента; Парзеновская оценка; сегмент; функция плотности.

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