UDC 517

SCOPUS CODE 2201

https://doi.org/10.36073/1512-0996-2024-1-176-181

Investigation of Epicycloid Mechanism Dynamics

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Abstract.

A dynamic analysis of a planetary mechanism with a satellite in external engagement is presented, where the central wheel is fixed. The trajectory of a driven link of such type of mechanism allows it to use it in machines, which provides technological processes where the executive or output link carries out pulsating motion which is required during of stop of the driven link. Mechanisms of this type mainly are used in the textile industry, where the processing of a material requires the actuator to be able to make minor fluctuations with respect to the temporal position. Here mentioned, determines the necessity of solution not only problems of kinematics but also here given system's dynamic analysis, which would be used during design of such epicycloid mechanisms. Based on the mentioned, below is given the methodology of calculation of dynamic parameters of epicycloid mechanism.

Keywords: cycloid; force; mechanism; movement; satellite; work.

Introduction

Planetary gears with satellites can be used for different purposes, among them it is necessary to mention sawing machines, where the output link must fulfill small oscillating movement in order to penetrate the needles in dense materials easily. For the mentioned reason, some machines used in light industry for obtaining quasi-stop motion of the executive mechanism the output link, are widely used in textile and light industry machines. For example, in special sewing machines, there are mechanisms for getting small transverse or longitudinal displacement of the needle during the stitching of dense tissues. In such cases, it is necessary to use mechanisms with less kinematic complexity and making it possible to develop as high speeds of executive mechanisms as possible. For development of here said goal bellow is considered the scheme of planetary mechanism (see Fig.1), where the planetary gear is in external engagement with a stationary central wheel, where the small oscillations of needles are provided by point K linked to satellite.

Main Part

Our task is to determine the driving moment that acting on the input link *OA* to ensure a uniform speed of the point *K,* which rotating of the sewing thread of the driven link, that moves on the basis of given forces acting on the mechanism.

This mechanism has one degree of freedom, since the angle of rotation φ of the crank OA determines the position of all points of the mechanism. As a generalized coordinate, we choose the angle φ counted from the horizontal axis in counterclockwise.

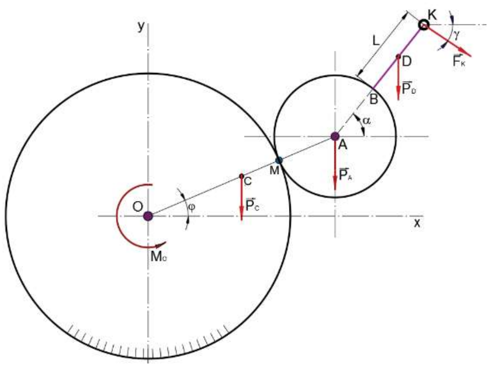

Рис. 1

In order to determine the dynamic parameters let's use the Lagrange equation, that for variable φ can be written as:

$$\frac{d}{dx}\cdot\frac{\partial T}{\partial \varphi}-\frac{\partial T}{\partial \varphi}=Q_{\varphi}. \qquad (1)$$

The active forces are P_C – crank weight, P_A – wheel weight *2,* P_D – rod weight *3,* M_O – torque applied to the crank OA. F_K – force applied at the end of the rod *AK* from the side of the thread. All connections imposed on the system are ideal.

Let us give to the crank OA a possible angular displacement δφ in the direction of increasing angle φ, i.e. counterclockwise. In order to determine the generalized force $Q\varphi$ we calculate the sum of the works active on the possible displacement $\delta\varphi$.

$$\delta A=\delta A_O+\delta A_C+\delta A_A+\delta A_D+\delta A_K. \qquad (2)$$

In order to solve the given equation, it is necessary from the beginning to determine the works done by the forces P_C, P_A, P_D applied in the centers of gravity of *1, 2, 3*, links and also the force F_K applied to the point at the end of the rod 3 acting from the side of the stretched thread. From the beginning, it is necessary to write the values of the elementary possible displacements of the points of application of forces depending on the angle of rotation of the crank *OA* and for the point *A* we will have:

$$\delta\varphi\cdot|OA|=\delta S_A. \qquad (3)$$

Rotation of the link 2 relatively to the point M will be:

$$\delta\beta=\frac{\delta S_A}{r_2}=\frac{\delta\varphi\cdot(r_1+r_2)}{r_2} \qquad (4)$$

The distance from the point of contact of links 1 and 2 to the center of gravity of the rod D and to the end point K is determined by the law of cosines:

$$\cos(180^0-(\alpha-\varphi))=-\cos(\alpha-\varphi), \qquad (5)$$

and meaning of the distances *MD* and *MK* can be written as:

$$|MD|=\sqrt{r_2^2+|AD|^2+r_2\cdot|AD|\cdot cos(\alpha-\varphi)}\,; \qquad (6)$$

$$|MK|=\sqrt{r_2^2+|AK|^2+r_2\cdot|AK|\cdot cos(\alpha-\varphi)}\,. \qquad (7)$$

At the same time, the inclination angle β can be determined from the sine theorem, as:

$$\frac{|MD|}{sin(\alpha-\varphi)}=\frac{|AD|}{sin\beta}; \qquad (8)$$

Based on the given relation the meaning of angle β can be determined as:

$$\beta = (-1)^k arcsin\frac{|AD|\cdot sin(\alpha-\varphi)}{|MD|} + \pi k;$$
$$k = 1,2,3, \dots; \qquad (9)$$

If we take in account, that the angle between the force $\vec{P}_D$ and the elementary displacement $\delta\vec{S}_D$ is $\widehat{\vec{P}_D,\ \delta\vec{S}_D} = \varphi + \beta + 180^{\circ}$, than the work done by the force $\vec{P}_D$ on the displacement $\delta\vec{S}_D$ can be written as:

$$\delta A_D = P_D \cdot \delta S_D \cdot cos(180^{\circ} + \varphi + \beta) = -P_D \cdot \delta S_D \cdot cos(\varphi + \beta). \qquad (10)$$

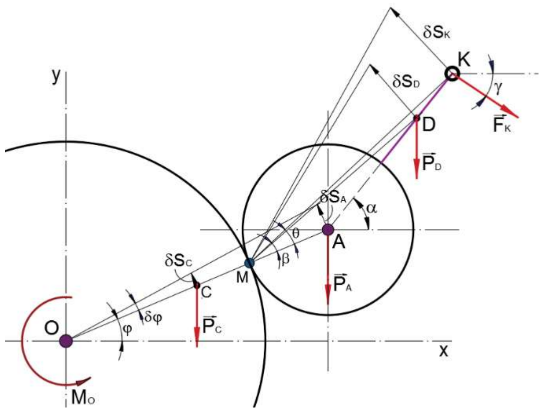

Рис. 2

In case when the angle among force $\vec{P}_K$ and the displacement $\delta\vec{S}_K$ can be calculated be following equality:

$$\widehat{\vec{F}_K,\ \delta\vec{S}_K} = \varphi + \theta + \gamma + 90^{\circ}. \qquad (11)$$

The work done by the force $\vec{P}_K$ on the displacement $\delta\vec{S}_K$ can be calculate by following formula:

$$\delta A_K = F_K \cdot \delta S_K \cdot cos(\varphi + \theta + \gamma + 90^{\circ}) = -F_K \cdot \delta S_K \cdot cos(\varphi + \theta + \gamma). \qquad (12)$$

Expressions for elementary displacements of points D and K depending on the increment δφ will have the following view:

$$\delta S_D = \frac{\delta S_A}{r_2} \cdot |MD| = \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot |MD| =$$
$$= \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot \sqrt{r_2^2 + |AD|^2 + r_2 \cdot |AD| \cdot cos(\alpha - \varphi)}\ ; \qquad (13)$$

$$\delta S_K = \frac{\delta S_A}{r_2} \cdot |MK| = \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot |MK| =$$
$$= \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot \sqrt{r_2^2 + |AK|^2 + r_2 \cdot |AK| \cdot cos(\alpha - \varphi)}\ . \qquad (14)$$

After that, we find the total virtual work of all the forces applied to the mechanism:

$$\delta A = M_O \cdot \delta\varphi - P_1 \cdot |OC| \cdot cos\varphi \cdot \delta\varphi - P_2 \cdot |OA| \cdot cos\varphi \cdot \delta\varphi -$$
$$-P_D \cdot \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot \sqrt{r_2^2 + |AD|^2 + r_2 \cdot |AD| \cdot cos(\alpha - \varphi)} \cdot cos(\varphi + \beta) - F_K \cdot \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2}$$
$$\cdot \sqrt{r_2^2 + |AK|^2 + r_2 \cdot |AK| \cdot cos(\alpha - \varphi)} \times cos(\varphi + \theta + \gamma) =$$
$$= M_O \cdot \delta\varphi - P_1 \cdot \frac{r_1 + r_2}{2} \cdot cos\varphi \cdot \delta\varphi - P_2 \cdot (r_1 + r_2) \cdot cos\varphi \cdot \delta\varphi -$$
$$-P_D \cdot \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot \sqrt{r_2^2 + \left(r_2 + \frac{l}{2}\right)^2 + r_2 \cdot \left(r_2 + \frac{l}{2}\right) \cdot cos(\alpha - \varphi)} \cdot cos(\varphi + \beta) -$$
$$-F_K \cdot \frac{\delta\varphi \cdot (r_1 + r_2)}{r_2} \cdot \sqrt{r_2^2 + (r_2 + l)^2 + r_2 \cdot (r_2 + l) \cdot cos(\alpha - \varphi)} \times$$
$$\times cos(\varphi + \theta + \gamma)\ . \qquad (15)$$

Considering that δA=Q_φ·δφ we find the generalized force corresponding to the generalized coordinate φ

ISSN 1512-0996
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$$Q_\varphi = M_O - P_1 \cdot \frac{r_1 + r_2}{2} \cdot cos\varphi - P_2 \cdot (r_1 + r_2) \cdot cos\varphi -$$

$$-P_D \cdot \frac{(r_1+r_2)}{r_2} \cdot \sqrt{r_2^2 + \left(r_2 + \frac{l}{2}\right)^2 + r_2 \cdot \left(r_2 + \frac{l}{2}\right) \cdot cos(\alpha - \varphi)} \cdot cos(\varphi + \beta) - F_K \cdot \frac{(r_1+r_2)}{r_2} \cdot$$

$$\cdot \sqrt{r_2^2 + (r_2 + l)^2 + r_2 \cdot (r_2 + l) \cdot cos(\alpha - \varphi)} \cdot cos(\varphi + \theta + \gamma). \quad (16)$$

We proceed to the calculation of the kinetic energy of the mechanism, which includes crank - 1, a gear -2 with a rod - 3.

$$T = T_1 + T_2 \,. \quad (17)$$

The kinetic energy of the crank OA rotating relatively of the fixed point located on the perpendicular to the plane that depicted on the figure, which can be calculated by the following formula:

$$T_1 = \frac{1}{2} \cdot I_O \cdot \dot{\varphi}^2. \quad (18)$$

Where: $I_O = \frac{1}{3} \cdot \frac{P_1}{g} \cdot (r_1 + r_2)^2$ is the crank's moment of inertia and hence expression for kinetic energy can be written as:

$$T_1 = \frac{1}{6} \cdot \frac{P_1}{g} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 \,. \quad (19)$$

The kinetic energy of a gear wheel in a place with a rod that makes a plane movement is equal to:

$$T_2 = \frac{1}{2} \cdot \frac{P_2}{g} \cdot v_A^2 + \frac{1}{2} \cdot I_A \cdot \omega_2^2 \;; \quad (20)$$

Here V_A is the speed of point A, which is the end of the crank OA. The value of this speed can be calculated as:

$$V_A = |OA| \cdot \dot{\varphi} = (r_1 + r_2) \cdot \dot{\varphi} \,. \quad (21)$$

Now let's find the speed of the same point belonging to the gear wheel in relation to the instantaneous center of speed M of the wheel:

$$V_A = |AM| \cdot \omega_2 = r_2 \cdot \omega_2; \quad (22)$$

Comparing formulas (8) and (9) we find:

$$\omega_2 = \frac{(r_1+r_2)\cdot\dot{\varphi}}{r_2} \,. \quad (23)$$

At the same time, the moment of inertia of the gear wheel 2 with the rod 3 can be calculated by the expression:

$$I_A = \left\{\frac{P_2 \cdot r_2^2}{2g} + \frac{P_3}{g} \cdot \left[\frac{l^2}{12} + \left(r_2 + \frac{l}{2}\right)^2\right]\right\}, \quad (24)$$

and after setting the values of V_A, ω_2 and I_A in formula (7), we will have:

$$T_2 = \frac{1}{2} \cdot \frac{P_2}{g} \cdot V_A^2 + \frac{1}{2} \cdot I_A \cdot \omega_2^2 = \frac{1}{2} \cdot \frac{P_2}{g} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 + \frac{1}{2}\left\{\frac{P_2 \cdot r_2^2}{2g} + \frac{P_3}{g} \cdot \left[\frac{l^2}{12} + \left(r_2 + \frac{l}{2}\right)^2\right]\right\} \cdot \frac{(r_1+r_2)^2 \cdot \dot{\varphi}^2}{r_2^2}. \quad (25)$$

Based on here given expressions can be written the formula for determination of the kinetic energy of the whole mechanism:

$$T = T_1 + T_2 = \left\{\frac{2P_1+9P_2}{12g} + \frac{P_3}{2g \cdot r_2^2} \cdot \left[\frac{l^2}{12} + \left(r_2 + \frac{l}{2}\right)^2\right]\right\} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 \,. \quad (26)$$

Let us calculate the partial derivative of the kinetic energy with respect to the generalized velocity $\dot{\varphi}$:

$$\frac{\partial T}{\partial \dot{\varphi}} = \left\{\frac{2P_1+9P_2}{6g} + \frac{P_3}{g \cdot r_2^2} \cdot \left[\frac{l^2}{12} + \left(r_2 + \frac{l}{2}\right)^2\right]\right\} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi} \,. \quad (27)$$

Since the kinetic energy of the system does not depend on the generalized coordinate φ, we can write:

$$\frac{\partial T}{\partial \varphi} = 0. \quad (28)$$

After differentiation with respect to time, expression (27) will have:

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{\varphi}} = \left\{\frac{2P_1+9P_2}{6g} + \frac{P_3}{g \cdot r_2^2} \cdot \left[\frac{l^2}{12} + \left(r_2 + \frac{l}{2}\right)^2\right]\right\} \cdot (r_1 + r_2)^2 \cdot \ddot{\varphi} \,. \quad (29)$$

In expanded form equation (1) will be written as:

$$\left\{\frac{2P_1+9P_2}{6g}+\frac{P_3}{g\cdot r_2^2}\cdot\left[\frac{l^2}{12}+\left(r_2+\frac{l}{2}\right)^2\right]\right\}\cdot(r_1+r_2)^2\cdot\ddot{\varphi}=M_O-P_1\cdot|OC|\cdot cos\varphi-P_2\cdot|OA|\times$$
$$\times\cos\varphi-P_D\cdot\frac{(r_1+r_2)}{r_2}\cdot\sqrt{r_2^2+|AD|^2+r_2\cdot|AD|\cdot cos(\alpha-\varphi)}\cdot cos(\varphi+\beta)-$$
$$-F_K\cdot\frac{(r_1+r_2)}{r_2}\cdot\sqrt{r_2^2+|AK|^2+r_2\cdot|AK|\cdot cos(\alpha-\varphi)}\times cos(\varphi+\theta+\gamma). \quad (30)$$

From this expression we can obtain the angular acceleration of the link OA

$$\ddot{\varphi}=\frac{M_O-P_1\cdot|OC|\cdot cos\varphi-P_2\cdot|OA|\cdot cos\varphi}{\left\{\frac{2P_1+9P_2}{6g}+\frac{P_3}{g\cdot r_2^2}\cdot\left[\frac{l^2}{12}+\left(r_2+\frac{l}{2}\right)^2\right]\right\}\cdot(r_1+r_2)^2}-\frac{P_D\cdot\frac{(r_1+r_2)}{r_2}\cdot cos(\varphi+\beta)\cdot\sqrt{r_2^2+|AD|^2+r_2\cdot|AD|\cdot cos(\alpha-\varphi)}}{\left\{\frac{2P_1+9P_2}{6g}+\frac{P_3}{g\cdot r_2^2}\cdot\left[\frac{l^2}{12}+\left(r_2+\frac{l}{2}\right)^2\right]\right\}\cdot(r_1+r_2)^2}-$$
$$-\frac{F_K\cdot\frac{(r_1+r_2)}{r_2}\cdot cos(\varphi+\theta+\gamma)\sqrt{r_2^2+|AK|^2+r_2\cdot|AK|\cdot cos(\alpha-\varphi)}}{\left\{\frac{2P_1+9P_2}{6g}+\frac{P_3}{g\cdot r_2^2}\cdot\left[\frac{l^2}{12}+\left(r_2+\frac{l}{2}\right)^2\right]\right\}\cdot(r_1+r_2)^2} \quad (31)$$

If the link rotates uniformly, then $\ddot{\varphi}$ in this equation is equal to zero and the torque required for this will be:

$$M_O=P_1\cdot\frac{(r_1+r_2)}{2}\cdot cos\varphi+P_2\cdot(r_1+r_2)\cdot cos\varphi+$$
$$+P_D\cdot\frac{(r_1+r_2)}{r_2}\cdot cos(\varphi+\beta)\cdot\sqrt{r_2^2+|AD|^2+r_2\cdot|AD|\cdot cos(\alpha-\varphi)}+$$
$$+F_K\cdot\frac{(r_1+r_2)}{r_2}\cdot cos(\varphi+\theta+\gamma)\cdot\sqrt{r_2^2+|AK|^2+r_2\cdot|AK|\cdot cos(\alpha-\varphi)}\ . \quad (32)$$

Conclusion

It is no exaggeration that now cycloid mechanisms have firmly taken their position in the light and textile industries as a guiding mechanism for the sewing needle in special sewing machines that are designed to work in various fabrics when sewing zigzag and other complex types of stitches are necessary to obtain. These mechanisms allow slight fluctuations of the needle when it is inside the material being processed, and in this way, it prevents its damage during sewing. In this paper, a dynamic study of the movement of one of these mechanisms is given, an equation for the driving moment of the mechanism acting on the input link is found, which provides a uniform angular velocity of the executive body of this mechanism.

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UDC 517
SCOPUS CODE 2201
https://doi.org/10.36073/1512-0996-2024-1-176-181

ეპიციკლოიდური მექანიზმის დინამიკის კვლევა

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ანოტაცია. სტატიაში წარმოდგენილია სატელიტის მქონე პლანეტარული მექანიზმის დინამიკური კვლევა, რომელიც იმყოფება გარე მოდებაში უძრავ ცენტრალურ კბილა თვალთან. მექანიზმის ტრაექტორიის თავისებურებები საშუალებას იძლევა ეს მექანიზმი გამოყენებულ იქნეს ისეთ მანქანაში, რომელიც უზრუნველყოფს ტექნოლოგიურ პროცესებს, რომელთათვისაც ხანდახან საჭიროა მიმყოლი რგოლის მცირე რხევები გაჩერების ეტაპებზე. ასეთი ტიპის მექანიზმები გამოიყენება ტექსტილის წარმოებაში, სადაც რომელიმე მასალის დამუშავებისას მოითხოვება შემსრულებელი ორგანოს მცირე რხევების დაშვება მექანიზმის დროებითი გაჩერებისას.

საკვანძო სიტყვები: გადაადგილება; მექანიზმი; მუშაობა; სატელიტი; ციკლოიდა; ძალა.

The date of review 22.10.2023
The date of submission 30.10.2023
Signed for publishing 22.03.2024