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Study of the Dynamics of the Epicyclic Mechanism in the Presence of Sliding Friction and Rolling Friction

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Abstract. The article presents a dynamic solution of an epicyclic gear mechanism that is in external engagement with a fixed inner wheel. The geometric features of the trajectory of this mechanism make it possible to apply this mechanism in machines that provide technological processes, which sometimes require minor vibrations of the slave link during the stop stage. Mechanisms of this type are used in the textile industry, where when processing a material, it is required that the executive body can make minor fluctuations relative to the temporary position.

Keywords: Acceleration; Cycloid; Gear wheel; Mechanism; Moment; Speed.

Introduction

The article discusses the dynamic calculation of an epicyclic planetary mechanism when its satellite is in external engagement with a fixed internal support wheel. The design features of the mechanism allow this mechanism to be used in machines that provide technological processes, which sometimes require minor vibrations of the driven link during the stop stage. Mechanisms of this type are used in the textile industry, where the processing of the material requires that the actuator can make minor fluctuations relative to the temporary position. In this work, a study of the dynamics of such a mechanism is carried out for the case when sliding friction forces act on the driven wheel in the joint of the joint with the tip and the rolling friction moment at the point of contact of the movable wheel (satellite) with the fixed support inner wheel. As a result, formulas are derived to determine the acceleration of the driving link and the torque applied to this link during its uniform movement.

Main Part

Crank OA rotates under action of moment applied to point O -M_0, uniform wheel 2 with mass m_2 is driven through crank, which rolls on external surface of wheel- 1. It is necessary to determine the acceleration obtained by the crank under the action of external forces applied to the mechanism and friction forces applied in the hinge A gear 2 and rolling friction moment acting at the point of contact of this wheel with fixed wheel 1. The mechanism has one degree of freedom, i.e. its position is determined by one generalized coordinate.

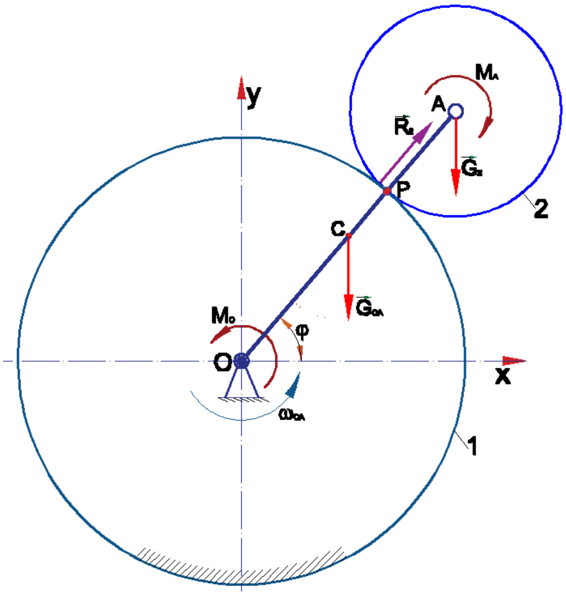

Fig. 1

To solve the problem, you need to use the Lagrange equation of the second kind.

The Lagrange equation for a generalized $\varphi -$ coordinate is:

$$\frac{d}{dt}\cdot\frac{\partial T}{\partial \dot{\varphi}} - \frac{\partial T}{\partial \varphi} = Q_\varphi \qquad (1)$$

Active forces are $G_{OA} = m_{OA}\cdot g -$ crank gravity, $G_2 = m_2 \cdot g -$ wheel 2 gravity, $M_O -$ moment applied to crank OA, moment of resistance forces acts in hinge A M_A reaction at the point of contact of wheels R_2 which creates rolling friction moment whose coefficient f .

To determine the generalized force, we calculate the sum of the works of all the listed forces acting on the mechanism at an elementary possible movement $\delta\varphi$, on which the crank imaginatively turns OA towards increasing angle φ.

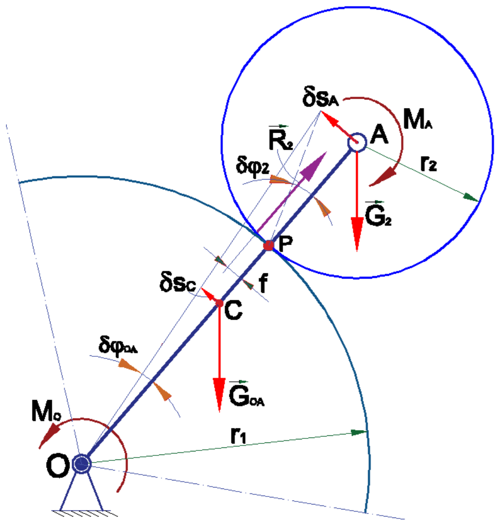

Fig. 2

After that, the reaction force from the stationary wheel can be calculated:

$$R_2 = G_2 \cdot \cos(90^\circ - \varphi) = G_2 \cdot \sin\varphi \qquad (2)$$

Let's calculate the possible angle of rotation of the wheel 2

$$\delta\varphi_2 = \frac{\delta s_A}{r_2} = \frac{(r_1 + r_2)\cdot\delta\varphi}{r_2} \qquad (3)$$

The expression to work with looks like this:

$$\delta A_{\text{кач}} = V \cdot \delta r_P + M_{zP}\cdot\delta\varphi_2 \qquad (4)$$

If we move the pole of bringing forces to a point P, we will have:

$$\delta r_P = 0 \qquad (5)$$

$$\delta A_{\text{кач}} = M_{zP}\cdot\delta\varphi_2 \qquad (6)$$

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The total moment of all forces acting on the wheel 2 relative to the point P will be:

$$M_{zP} = -R_2 \cdot f - M_A - G_2 \cdot r_2 \cdot \cos\varphi =$$
$$= -M_A - G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \quad (7)$$

After that, we find work on the possible movement of this moment:

$$\delta A_1 = M_O \cdot \delta\varphi - G_{OA} \cdot \delta s_C \cdot cos\varphi - G_2 \cdot \delta s_A \cdot \cos\varphi =$$
$$= M_O \cdot \delta\varphi - G_{OA} \cdot \frac{r_1 + r_2}{2} \cdot cos\varphi\delta\varphi -$$
$$-G_2 \cdot (r_1 + r_2) \cdot cos\varphi\delta\varphi \quad (8)$$
$$\delta A_2 = [-M_A - G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f)] \cdot \delta\varphi_2 =$$
$$= [-M_A - G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f)] \cdot \frac{(r_1+r_2)\cdot\delta\varphi}{r_2} \quad (9)$$

Given this, the sum of the work of all the forces applied to the mechanism on elementary possible movements will be determined using the expression:

$$\delta A = \delta A_1 + \delta A_2 = M_O \cdot \delta\varphi -$$
$$-G_{OA} \cdot \frac{r_1+r_2}{2} \cdot cos\varphi\delta\varphi - G_2 \cdot (r_1 + r_2) \cdot cos\varphi\delta\varphi +$$
$$-M_A \cdot \frac{(r_1 + r_2) \cdot \delta\varphi}{r_2} -$$
$$-G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \cdot \frac{(r_1+r_2)\cdot\delta\varphi}{r_2} \quad (10)$$

Hence, let's take the generalized force by dividing this expression by $\delta\varphi$, then we get:

$$Q_\varphi = M_O - G_{OA} \cdot \frac{r_1 + r_2}{2} \cdot cos\varphi - G_2 \cdot (r_1 + r_2) \cdot cos\varphi$$
$$- M_A \cdot \frac{(r_1 + r_2)}{r_2} -$$
$$-G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \cdot \frac{(r_1+r_2)}{r_2} \quad (11)$$

Based on the fact that the mechanism includes movable links, a crank OA and a gear wheel 2, the total kinetic energy is determined by the formula:

$$T = T^{(1)} + T^{(2)} \quad (12)$$

The crank OA rotates around the axis O and its kinetic energy is determined by the formula:

$$T^{(1)} = \frac{1}{2} J_O \dot{\varphi}^2 \quad (13)$$

Where

$$J_O = \frac{1}{3} m_{OA} \cdot OA^2 = \frac{1}{3} m_{OA} \cdot (r_1 + r_2)^2 \quad (14)$$

There is a moment of inertia of the crank and after its substitution in formula (14)

Let's receive:

$$T^{(1)} = \frac{1}{6} m_{OA} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 \quad (15)$$

The kinetic energy of a gear that makes a flat motion is:

$$T^{(2)} = \frac{1}{2} m_2 v_A^2 + \frac{1}{2} J_2 \omega_2^2 \quad (16)$$

Here you first need to determine the speed of the center of rotation of the wheel, which is the end of the crank OA:

$$v_A = OA \cdot |\dot{\varphi}| = (r_1 + r_2) \cdot |\dot{\varphi}| \quad (17)$$

To determine the angular speed of link 2, consider the speed of the same point A belonging to the gear with respect to the instantaneous center of speed P of the wheel

$$v_A = AP \cdot \omega_2 = r_2 \cdot \omega_2 \quad (18)$$

From this we get:

$$\omega_2 = \frac{(r_1+r_2)}{r_2} \cdot |\dot{\varphi}| \quad (19)$$

The moment of inertia of the gear 2 wheel is calculated by the formula:

$$J_A = \frac{1}{2} m_2 \cdot r_2^2 \quad (20)$$

The complete expression for the kinetic energy of the wheel is written as follows:

$$T^{(2)} = \frac{1}{2} m_2 \cdot \left((r_1 + r_2) \cdot |\dot{\varphi}|\right)^2 + \frac{1}{4} m_2 \cdot r_2^2 \cdot \omega_2^2 =$$
$$= \frac{1}{2} m_2 \cdot \left((r_1 + r_2) \cdot |\dot{\varphi}|\right)^2 + \frac{1}{4} m_2 \cdot r_2^2$$
$$\cdot \left(\frac{(r_1 + r_2)}{r_2} \cdot |\dot{\varphi}|\right)^2 =$$
$$= \frac{1}{2} m_2 \cdot (r_1 + r_2)^2 |\dot{\varphi}|^2 + \frac{1}{4} m_2 \cdot (r_1 + r_2)^2 |\dot{\varphi}|^2 =$$
$$= \frac{3}{4} m_2 \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 \quad (21)$$

After substituting expression (16) and (22) into formula (13), we obtain an expression describing the complete kinetic energy of the mechanism:

$$T = T^{(1)} + T^{(2)} = \frac{1}{6} m_{OA} \cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 + \frac{3}{4} m_2$$
$$\cdot (r_1 + r_2)^2 \cdot \dot{\varphi}^2 =$$

$$= \left(\frac{1}{6}m_{OA} + \frac{3}{4}m_2\right)(r_1 + r_2)^2 \cdot \dot{\varphi}^2 =$$

$$= \frac{2m_{OA}+9m_2}{12}(r_1 + r_2)^2 \cdot \dot{\varphi}^2 \qquad (22)$$

After that, we calculate the partial derivative of kinetic energy T from the generalized speed $\dot{\varphi}$:

$$\frac{\partial T}{\partial \dot{\varphi}} = \frac{2m_{OA}+9m_2}{6}(r_1 + r_2)^2 \cdot \dot{\varphi} \qquad (23)$$

And take the derivative of the obtained result in time:

$$\frac{d}{dt} \cdot \frac{\partial T}{\partial \dot{\varphi}} = \frac{2m_{OA}+9m_2}{6}(r_1 + r_2)^2 \cdot \ddot{\varphi} \qquad (24)$$

Note that the total kinetic energy of the system expressed by formula (23) does not depend on the generalized coordinate φ, based on which we get:

$$\frac{\partial T}{\partial \varphi} = 0 \qquad (25)$$

After substituting the expressions (12), (25) and (26) into the Lagrange equation (1), we obtain the differential equation of mechanism motion for the generalized coordinate φ:

$$\frac{2m_{OA}+9m_2}{6}(r_1 + r_2)^2 \cdot \ddot{\varphi} =$$

$$= M_O - G_{OA} \cdot \frac{r_1+r_2}{2} \cdot cos\varphi - G_2 \cdot (r_1 + r_2) \cdot cos\varphi -$$

$$-M_A \cdot \frac{(r_1+r_2)}{r_2} - G_2 \cdot (r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \cdot \frac{(r_1+r_2)}{r_2} \qquad (26)$$

Where the crank acceleration is determined OA

$$\ddot{\varphi} = \frac{M_O}{\frac{2m_{OA}+9m_2}{6}(r_1 + r_2)^2} - \frac{3G_{OA}cos\varphi}{(2m_{OA}+9m_2)(r_1 + r_2)}$$

$$- \frac{6G_2 \cdot cos\varphi}{(2m_{OA}+9m_2)(r_1 + r_2)} -$$

$$- \frac{6M_A}{(m_{OA}+9m_2)(r_1 + r_2)r_2} -$$

$$-(r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \cdot \frac{6G_2}{(m_{OA}+9m_2)(r_1+r_2)r_2} \qquad (27)$$

Uniform rotation of the crank occurs under the condition:

$$M_O = \frac{1}{2}G_{OA}cos\varphi \cdot (r_1 + r_2) + G_2 \cdot cos\varphi(r_1 + r_2) +$$

$$+ \frac{M_A(r_1+r_2)}{r_2} + (r_2 \cdot \cos\varphi + \sin\varphi \cdot f) \cdot \frac{G_2(r_1+r_2)}{r_2} \qquad (28)$$

Conclusion

The article discusses the dynamic solution of a hypocyclic planetary mechanism when its satellite is in external engagement with a fixed inner wheel. Design features make it possible to use this mechanism in machines that provide technological processes, which sometimes require minor vibrations of the slave link during the shutdown stage. Mechanisms of this type are used in the textile industry, where when processing material it is required that the executive body can make minor fluctuations relative to the temporary position. In this work, a study of the dynamics of such a mechanism is carried out and as a result, formulas are derived to determine the acceleration of the driving link and the torque applied to this link during its uniform movement.

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