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Formulation of a One-Dimensional Equation for the Mudflow Process Caused by a Dam Failure in a Pond Under Conditions of Nonuniform and Uneven Flow Motion

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Abstract.

The two approximate theories used in mathematical modeling of wave processes in reservoirs: the shallow water (long-wave) theory and the small-amplitude wave theory. In shallow water theory, it is assumed that the water depth is small compared to other characteristic quantities (in particular, the wavelength). The main assumption is that the magnitude of the water velocity component in the vertical direction is negligible; accordingly, the pressure distribution in the water submits to depth follows the hydrostatic law, and wave motion is represented using one-dimensional (1D) or two-dimensional (2D) (horizontal) models.

The purpose of this technical article is to develop a method for calculating the characteristics of a high velocity, long, nonuniform extreme mudflow wave generated by a potential earthen dam failure in a narrow mining pond using one-dimensional (1D) shallow water theory.

Keywords: Bottom; Dam; Flood; Flow; Pond; Riverbed; Wave.

Introduction

A possible failure of an earthen dam in a narrow mining pond will result in the emergence of a high (relative to the depth of the reservoir), long, nonuniform mudflow wave moving at high velocity.

Since in this case the vertical and transverse components of the velocity are practically equal to zero with respect to the longitudinal (along the channel) velocity, it is best to calculate the characteristics of such extreme waves using one-dimensional (1D) shallow water theory.

Main Part

When solving this problem, higher results in assessing and predicting the risks of potential hazards obtained if the actual terrain forms (the cross-section size should correspond to the wave height) and roughness are considered, which is individual for each valley. (Fig. 1)

The equation has the following form:

One-dimensional (1D) shallow water system of equations.

$$\begin{gathered}\frac{\partial}{\partial t}u+\frac{\partial}{\partial x}\left(\frac{u^2}{2}+gH\right)+g\left(S_f-S_g\right)=0\\ \frac{\partial}{\partial t}H+\frac{\partial}{\partial x}(Hu)+\left(\frac{A}{B}-H\right)\frac{\partial u}{\partial x}+\frac{u}{B}\left(\frac{\partial A}{\partial x}\right)^{H=const}=0\end{gathered} \tag{1}$$

Where:

$H=H(x,t)$	- Flow depth;
$A=A(x,H)$	- Wetted cross-sectional area;
$B=B(x,H)$	- Width of wetted cross-sectional surface;
$u=u(x,t)$	- Flow velocity;
$Q=Au$	- Flow rate;
$S_f=\frac{n^2}{\sqrt[3]{R}}\cdot\frac{u\lvert u\rvert}{R}=n^2\frac{u\lvert u\rvert}{R^{4/3}}$	- Hydraulic gradient;
$R=\frac{A}{P_A}$	- Hydraulic radius;
P_A	- Hydraulic perimeter;
$S_g=-\frac{\partial}{\partial x}Z$	- Bottom gradient
$Z=Z(x)$	- Bottom height

Note that the third term of the obtained continuity equation characterizes the real cross-section of the terrain, that is, the deviation of the cross-section shape from the shape of a rectangle, and $\left(\frac{A}{B}-H\right)\cong 0$ for a sufficiently large width of the cross-section compared to the depth of the rectangular shape.

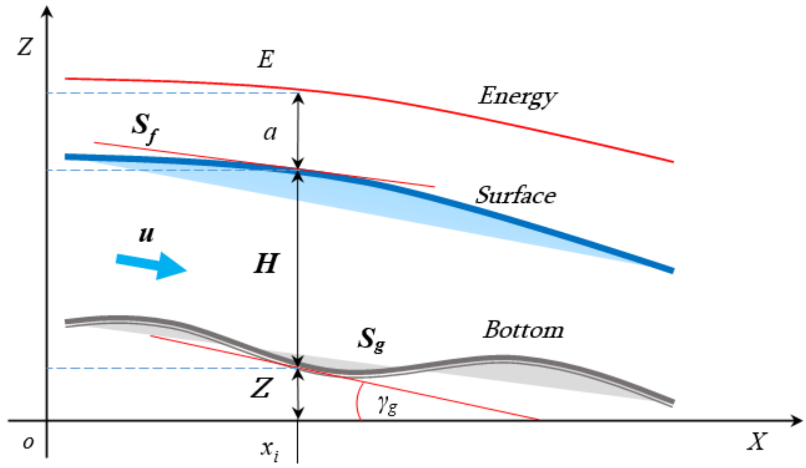

Figure 1. Calculation scheme for mudflow caused by dam failure

Similarly, the fourth term of the continuity equation characterizes the uniformity of the cross-section along the length and $\frac{1}{B}\left(\frac{\partial A}{\partial x}\right)^{H=const} \cong 0$ for a small change in the cross-section of the riverbed and a sufficiently large cross-section width compared to the depth, in the case of same depth.

The characteristics of the cross-section are determined according to topological data:

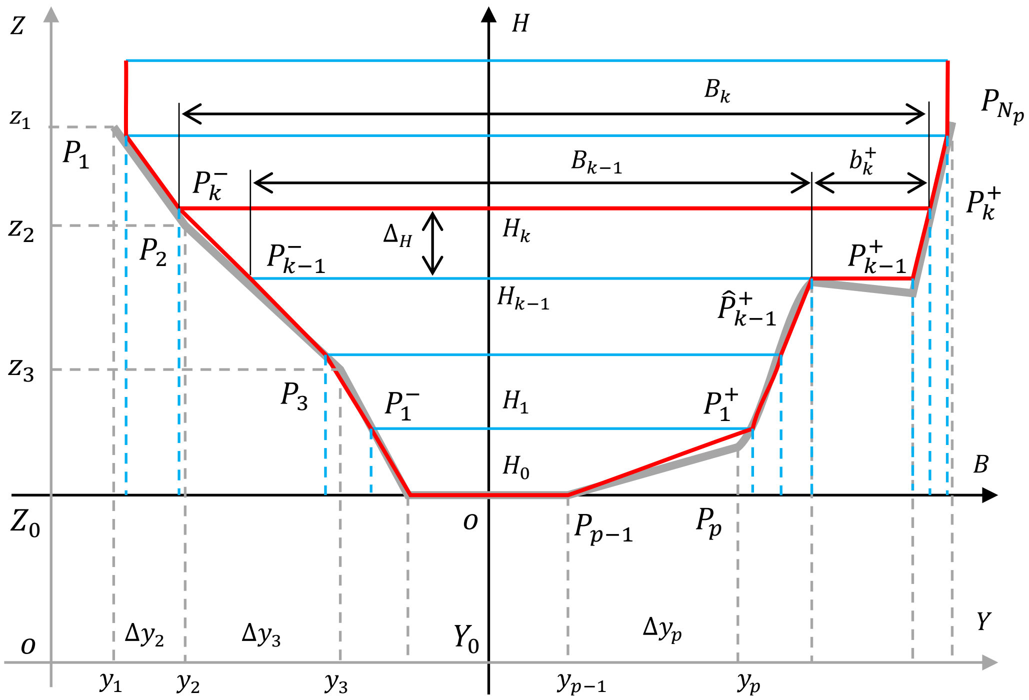

Figure 2. Cross-section approximation and characteristics calculation scheme

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To obtain a numerical representation, discretization is performed along the entire length of the riverbed, including the pond and dam areas, and the approximate characteristics of the real cross-sections are determined (Fig. 2). Without limitation of generality, the initial data for each cross-section is the $P_p(\Delta y_p, z_p)$ number of benchmarks, which determines the location of a point in the vertical plane and the distance from the previous benchmark, and can be assumed that $\Delta y_1 = 0$.

With the formulas discussed, it is possible to pre-calculate the discrete values of H_k, A_k, B_k, P_k, R_k in each cross-section and then, if necessary, obtain continuous values by approximation for any *H*.

Consider the linear approximation method. It is possible to find for the intersection parameters k, σ_H such that

$$\begin{gathered} H_{k-1} \le H < H_k \\ \sigma_H = \frac{H - H_{k-1}}{H_k - H_{k-1}} = \frac{1}{\Delta_H}(H - H_{k-1}) \\ 0 \le \sigma_H < 1 \end{gathered} \tag{2}$$

Accordingly, the continuous values determined by the proportionality coefficient σ_H can be calculated for H as follows:

$$\begin{aligned} B(H) &= B_H = B_{k-1} + \sigma_H (B_k - B_{k-1}) \\ P(H) &= P_H = P_{k-1} + \sigma_H (P_k - P_{k-1}) \\ A(H) &= A_H = A_{k-1} + \frac{\sigma_H \Delta_H}{2}(B_H + B_{k-1}) \\ R(H) &= R_H = \frac{A_H}{P_H} \end{aligned} \tag{3}$$

In addition to normalizing each cross-section with respect to *H*, it is also necessary to normalize with respect to the length L with a step of Δ_L (Fig. 3).

The methodology we have considered involves linear approximation of not the cross-section, but only the normalized parameters in the *L*-section, which significantly simplifies the task and calculations. The method consists in the following: divide the oL axis into Δ_L parts and for each L cross-section find the closest left and right pair S_{s-1}, S_s (where S denotes the index of the predefined, initial intersections), corresponding to the distances L_{s-1}, L_s:

$$\begin{gathered} L_{s-1} \le L < L_{s+1} \\ \sigma_L = \frac{H - L_{s-1}}{L_S - L_{s-1}} = \frac{1}{\Delta_L}(H - L_{s-1}) \\ 0 \le \sigma_L < 1 \end{gathered} \tag{4}$$

Let denote the functions A_H, B_H, P_H, R_H in the cross-section S by $A_{H,s}, B_{H,s}, P_{H,s}. R_{H,s}$. Accordingly, the continuous values determined by the proportionality coefficient σ_L for H can be calculated as follows:

$$\begin{aligned} B(L,H) &= B_{LH} = B_{H,s-1} + \sigma_L (B_{H,s} - B_{H,s-1}) \\ P(L,H) &= P_{LH} = P_{H,s-1} + \sigma_L (P_{H,s} - P_{H,s-1}) \\ A(L,H) &= A_{LH} = A_{H,s-1} + \sigma_L (A_{H,s} - A_{H,s-1}) \\ R(L,H) &= R_{LH} = \frac{A_{LH}}{P_{LH}} \end{aligned} \tag{5}$$

In general, the construction of cross-section benchmarks is carried out at all interested geographical locations, therefore, using the discussed formulas of the shallow water theory, it is possible to pre-calculate the discrete values of H_k, A_k, B_k, P_k, R_k in each cross-section, and then, if necessary, approximate continuous values for any *H*. Consider the linear approximation method. It is possible to find for the intersection parameters , σ_H H such that

$$\begin{gathered} H_{k-1} \le H < H_k \\ \sigma_H = \frac{H - H_{k-1}}{H_k - H_{k-1}} = \frac{1}{\Delta_H}(H - H_{k-1}) \\ 0 \le \sigma_H < 1 \end{gathered} \tag{6}$$

Accordingly, the continuous values determined by the proportionality coefficient σ_H can be calculated for H as follows:

$$\begin{aligned} B(H) &= B_H = B_{k-1} + \sigma_H (B_k - B_{k-1}) \\ P(H) &= P_H = P_{k-1} + \sigma_H (P_k - P_{k-1}) \\ A(H) &= A_H = A_{k-1} + \frac{\sigma_H \Delta_H}{2}(B_H + B_{k-1}) \\ R(H) &= R_H = \frac{A_H}{P_H} \end{aligned} \tag{7}$$

In addition to normalizing each cross-section concerning *H*, it is also necessary to normalize with respect to the length L with a step of Δ_L.

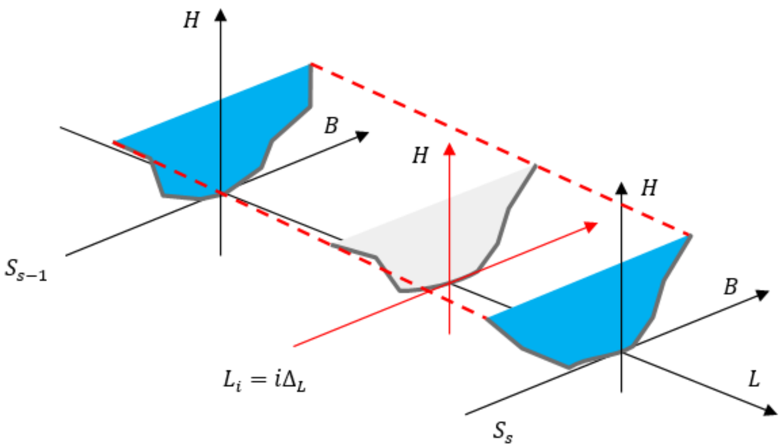

Figure 3. Calculation scheme for terrain approximation and characteristics

Based on the above, to solve ultimate value problems, it is sufficient to construct only a few initial critical cross-sections (where homogeneity decomposed or precise data is required) and, using the discussed method, calculate the necessary parameters obtained from continuous functions.

$$\begin{gathered} Z = Z(x,t), t \leq T_0 \\ A = A(x,H,t), t \leq T_0 \\ B = B(A,H) \\ P = P(A,H) \\ R = R(A,H) \end{gathered} \tag{8}$$

As for considering dam failure, either we assume an instantaneous failure to obtain maximum effect or we determine the functions of the bottom, cross-sectional area change, and other necessary characteristics over a short period of time in the dam's cross-section area during the time of dam failure, at the dam base section.

Conclusion

The formulas implement the changes in the cross-section due to the dam failure, but the failure itself and its mathematical modeling are not the subject of our research, therefore, we will limit ourselves to simplified formulas, according to which the cross-section and bottom height change by a certain value over a certain time period.

References

1. Iremashvili, Kh., & Berdzenashvili, G. (2016). The methodology of risk assessment and forecast of earth dam failure during high water in a deep mountain reservoir. In *VI International Scientific and Technical Conference "Modern-Day Problems of Water Economies, Environmental Protection, Architecture and Construction": Program and proceedings* (pp. 95–100).
2. Iremashvili, Kh., Berdzenashvili, G., & Jinjikhashvili, G. (2015). One-dimensional (1D) numerical modeling of the process of wave generation caused by flood inflow into a reservoir with complex terrain. *Construction*, (4[39]), 51–54.
3. Gvelesiani, T., Danelia, R., & Berdzenashvili, G. (1999). Mathematical model of unsteady hydrodynamic processes caused by mobile mud flow on the bottom of reservoir. *Bulletin of the Georgian Academy of Sciences, 159*(3), 457–459.
4. Gvelesiani, T. L. (2009). *The theory of wave generation as applied to problems of hydroecology*. Universal.

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