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Problem of Statics of the Linear Thermoelasticity of the Microstretch Materials with Microtemperatures for a Half-space

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Abstract. We consider the statics case of the theory of linear thermoelasticity with microtemperatures and microstrech materials. The representation formula of differential equations obtained in the paper is expressed by the means of four harmonic and four metaharmonic functions. These formulas are very convenient and useful in many particular problems for domains with concrete geometry. Here we demonstrate an application of these formulas to the III type boundary value problem for a half-space. Uniqueness theorems are proved. Solutions are obtained in quadratures.

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Key words: Fourier transform; function; microtemperature; microstretch; thermoelasticity.

Introduction. One of the basic methods solving three-dimensional problems of rigid deformable bodies is the Fourier method, which is based on the solution of differential equations of a given model by the method of separation of variables in a certain system of curvilinear coordinates. In case the construction of system of differential equations turns out complicated, its solution can be represented by a simple solution of Laplace and Helmholtz equations. Representations proposed by W.Kelvin, J.Hadamard, J.Boussinesq, M.papkovich, G.Neuber, E.Trefftz, G.Kolosow, N.Muskhelishvili and other authors are well known in the literature.

Mathematical models describing the chiral properties of the linear thermoelasticity with microtemperatures materials have been proposed by Iesan and recently it has been extended to a more general case, when the material points admit micropolar structure. In the representation and Fourier transforms we study the boundary value problems of statics of the thermoelasticity with microtemperatures for a half-space $x_3 > 0$. For a wider overview of the subject concerning different areas of applications we refer to the references J.Barber [1], M.Basheleishvili, L.Bitsadze [2], [3], D.Burchuladze, M.Kharashvili, K.Skhvitaridze [4], P.Cass, R.Quintanila [5], L.Giorgashvili, K.Skhvitaridze, M.Kharashvili [6], L.Giorgashvili, Sh.Zazashvili, R.Meladze [7], L. Giorgashvili, D.Metreveli [8], R.Kumar, T.Chadha [10], H.Sherief, H.Saleh [12], B.Singh, R.Kumar [13], K.Skhvitaridze, M.Kharashvili [14], M.Svanadze [15] and the references therein.

Main part

2. Basic equations and auxiliary theorems

The system of equations of statics in the linear theory of thermoelasticity of the microstretch materials with microtemperatures is written in the form

$$\mu\Delta u(x) + (\lambda+\mu)\mathrm{graddiv}u(x) + \eta\mathrm{grad}v(x) + \gamma\mathrm{grad}\theta(x) = 0, \tag{2.1}$$

$$\varkappa_6\Delta w(x) + (\varkappa_4+\varkappa_5)\mathrm{graddiv}w(x) - \varkappa_3\mathrm{grad}\theta(x) + \varkappa_2 w(x) = 0, \tag{2.2}$$

$$\varkappa\Delta\theta(x) + \varkappa_1\mathrm{div}w(x) = 0, \tag{2.3}$$

$$\eta_1\Delta v(x) - \eta\mathrm{div}u(x) - \varkappa_7\mathrm{div}w(x) + \gamma_1\theta(x) - \eta_2 v(x) = 0, \tag{2.4}$$

where Δ is three-dimensional Laplace operator, $u = (u_1, u_2, u_3)^\top$ is the displacement vector, $w = (w_1, w_2, w_3)^\top$ is the microtemperature vector, θ is the temperature measured from the constant absolute temperature T_0, $(T_0 > 0)$, v is the microstretch, $\top$ is the transposition symbol, $\lambda,\ \mu,\ \gamma,\ \gamma_1\ \eta,\ \eta_1,\ \varkappa,\ \varkappa_j,\ \ j = 1,2,\dots,7$ are constitutive coefficients, satisfying the conditions

$$\mu > 0,\ \ 3\lambda + 2\mu > 0,\ \ 3\varkappa_4 + \varkappa_5 + \varkappa_6 > 0,\ \ \varkappa_6 + \varkappa_5 > 0,\ \ \varkappa_6 - \varkappa_5 > 0,$$
$$(\varkappa_1 + T_0\varkappa_3)^2 < 4T_0\varkappa\varkappa_2,\ \ \gamma > 0,\ \ \eta_2(3\lambda+2\mu) - 3\eta^2 > 0,\ \ \eta_1 > 0$$

Assume that $U = (u, w, \theta, v)^\top$. The stress vector, which we denote by the symbol $P(\partial, n)U$, has the form

$$P(\partial,n)U = (P^{(1)}(\partial,n)U',\ P^{(2)}(\partial,n)W,\ P^{(3)}(\partial,n)U'',\ P^{(4)}(\partial,n)U''')^\top$$

where $U' = (u,\theta,v)^\top$, $U'' = (w,\theta)^\top$, $U''' = (w,v)^\top$, $n = (n_1,n_2,n_3)^\top$ is the unit vector,

$$P^{(1)}(\partial,n)U' = T^{(1)}(\partial,n)U' + (\eta v - \gamma\theta)n,$$
$$P^{(2)}(\partial,n)w = (\varkappa_5+\varkappa_6)\frac{\partial w}{\partial n} + \varkappa_4 n\mathrm{div}w + \varkappa_5[n\times\mathrm{rot}w],$$
$$P^{(3)}(\partial,n)U'' = \varkappa\frac{\partial\theta}{\partial n} + \varkappa_1(n\cdot w),$$
$$P^{(4)}(\partial,n)U''' = \eta_1\frac{\partial v}{\partial n} + \varkappa_7(n\cdot w),$$
$$T^{(1)}(\partial,n)u = 2\mu\frac{\partial u}{\partial n} + \lambda n\mathrm{div}u + \mu[n\cdot\mathrm{rot}u],$$

$\frac{\partial}{\partial n}$ is the derivative along the vector n. The symbol $(\cdot)$ and $[\times]$ denote the scalar and vector products of two vectors in $\mathbb{R}^3$.

Definition. The vector $U = (u, w, \theta)^\top$ is assumed to be regular in a domain $\Omega \subset \mathbb{R}^3$ if $U \in C^2(\Omega) \cap C^1(\overline{\Omega})$.

Theorem 1. *A vector* $U = (u, w, \theta, v)^\top$ *is a solution of system* (2.1)-(2.4) *in a domain* $\Omega \subset \mathbb{R}^3$, *if and only if it is represented in the form*

$$\begin{aligned} u(x) &= \alpha_1 c \tfrac{x}{|x|} + \operatorname{grad}\Phi_1(x) + \alpha_2 x_3 \operatorname{grad}\Phi_2(x) - e_3\Phi_2(x) + \\ &+ \operatorname{rot}(e_3\Phi_3(x)) - \alpha_3 x_3 \operatorname{grad}\Phi_4(x) + a_3 \operatorname{grad}\Phi_5(x) - \eta \operatorname{grad}\Phi_8(x), \\ w(x) &= \operatorname{grad}\left[2\alpha_4 \tfrac{c}{|x|} + \alpha_4 \tfrac{\partial\Phi_4(x)}{\partial x_3} + \alpha_5\Phi_5(x)\right] + \\ &+ \operatorname{rotrot}(e_3\Phi_6(x)) + \operatorname{rot}(e_3\Phi_7(x)), \\ \theta(x) &= 2\tfrac{c}{|x|} + \tfrac{\partial\Phi_4(x)}{\partial x_3} + (\lambda+2\mu)\lambda_1^2\Phi_5(x), \\ v(x) &= 2a_1 \tfrac{c}{|x|} + a_5 \tfrac{\partial\Phi_2(x)}{\partial x_3} + a_4 \tfrac{\partial\Phi_4(x)}{\partial x_3} + a_2\Phi_5(x) + (\lambda+2\mu)\lambda_3^2\Phi_8(x), \end{aligned} \tag{2.5}$$

where c *is arbitrary constant* $\Delta\Phi_j(x) = 0$,

$j = 1,2,3,4$ $\quad (\Delta - \lambda_1^2)\Phi_5(x) = 0, \quad (\Delta - \lambda_2^2)\Phi_j(x) = 0, \quad j = 6,7,$

$$(\Delta - \lambda_3^2)\Phi_8(x) = 0,$$

$$\begin{aligned} \lambda_1^2 &= \frac{\varkappa\varkappa_2 - \varkappa_1\varkappa_3}{\varkappa l_0} > 0, \quad l_0 = \varkappa_4 + \varkappa_5 + \varkappa_6, \quad \lambda_2^2 = \frac{\varkappa_2}{\varkappa_6} > 0, \\ \lambda_3^2 &= \frac{\eta_2(\lambda+2\mu) - \eta^2}{\eta_1(\lambda+2\mu)} > 0, \quad \alpha_1 = \frac{\gamma\eta_2 - \eta\gamma_1}{\eta_2(\lambda+2\mu) - \eta^2}, \quad \alpha_2 = \frac{\eta^2 - (\lambda+\mu)\eta_2}{\eta^2 - (\lambda+3\mu)\eta_2}, \\ \alpha_3 &= \frac{\eta_2\gamma - \eta\gamma_1}{\eta^2 - (\lambda+3\mu)\eta_2}, \quad \alpha_4 = -\frac{\varkappa_3}{\varkappa_2}, \quad \alpha_5 = -\frac{\varkappa(\lambda+2\mu)\lambda_1^2}{\varkappa_1}, \\ a_1 &= \frac{(\lambda+2\mu)\gamma_1 - \gamma\eta}{\eta_1(\lambda+2\mu)\lambda_3^2}, \quad a_2 = \frac{\lambda_1^2(\lambda+2\mu)}{\eta_1(\lambda_3^2 - \lambda_1^2)}\left(\frac{\gamma_1(\lambda+2\mu) - \gamma\eta}{\lambda+2\mu} + \frac{\varkappa\varkappa_7}{\varkappa_1}\lambda_1^2\right). \\ a_3 &= \gamma - \frac{\eta}{\lambda_1^2(\lambda+2\mu)}a_2, \quad a_4 = a_1 + \frac{\mu\eta\alpha_3}{\eta^2 - (\lambda+2\mu)\eta_2}, \quad a_5 = \frac{2\mu\eta}{\eta_2(\lambda+3\mu) - \eta^2} \end{aligned}$$

Proof. Assume that the vector $U = (u, w, \theta, v)^{\top}$ is a solution of system (2.1)-(2.4). From (2.2)-(2.3), we obtain

$$\Delta(\Delta - \lambda_1^2)\theta(x) = 0, \quad \lambda_1^2 = (\varkappa\varkappa_2 - \varkappa_1\varkappa_3)/\varkappa l_0 > 0,$$

from these equations we get

$$\theta(x) = 2\tfrac{c}{|x|} + \tfrac{\partial\Phi_4(x)}{\partial x_3} + (\lambda+2\mu)\lambda_1^2\Phi_5(x), \tag{2.6}$$

where c is arbitrary constant, $\Delta\Phi_4(x) = 0, \quad (\Delta - \lambda_1^2)\Phi_5(x) = 0.$

Substituting the value of $\theta(x)$ from (2.6) and $\operatorname{div}w(x) = -\frac{\varkappa}{\varkappa_1}\Delta\theta(x)$ into equation (2.2), we obtain

$$(\Delta - \lambda_2^2)w(x) = \frac{\chi_3}{\chi_6} grad\left(2\frac{c}{|x|} + \frac{\partial\Phi_4(x)}{\partial x_3}\right) + \frac{\chi}{\chi_1}(\lambda+2\mu)\cdot\lambda\left(\lambda_2^2 - \lambda_1^2\right) grad\Phi_5(x)$$

where $\lambda_2^2 = \varkappa_2/\varkappa_6 > 0.$

The solution of this equation has the form

$$\begin{aligned} 2w(x) &= \operatorname{grad}\left[2\alpha_4 \tfrac{c}{|x|} + \alpha_4 \tfrac{\partial\Phi_4(x)}{\partial x_3} + \alpha_5\Phi_5(x)\right] + \\ &+ \operatorname{rotrot}(e_3\Phi_6(x)) + \operatorname{rot}(e_3\Phi_7(x)) + \operatorname{grad}\Psi(x), \end{aligned} \tag{2.7}$$

where $(\Delta - \lambda_2^2)\Phi_j(x) = 0, \quad j = 6,7, (\Delta - \lambda_2^2)\Psi(x) = 0, \alpha_4 = -\varkappa_3/\varkappa_2, \alpha_5 = -\varkappa(\lambda+2\mu)\lambda_1^2/\varkappa_1.$

Substituting the values of $w(x)$ and $\theta(x)$, from (2.10) and (2.8) into (2.3), we obtain $\Psi(x) = 0$, thus

$$\begin{aligned} w(x) &= \operatorname{grad}\left[2\alpha_4 \tfrac{c}{|x|} + \alpha_4 \tfrac{\partial\Phi_4(x)}{\partial x_3} + \alpha_5\Phi_5(x)\right] + \\ &+ \operatorname{rotrot}(e_3\Phi_6(x)) + \operatorname{rot}(e_3\Phi_7(x)). \end{aligned} \tag{2.8}$$

If we apply the operator div to both part of equality (2.1), then we obtain

$$\Delta[(\lambda+2\mu)\operatorname{div}u(x) + \eta v(x) - \gamma\theta(x)] = 0.$$

From these equation, we get

$$(\lambda+2\mu)\mathrm{div}u(x)=\gamma\theta(x)-\eta v(x)+\eta_1(\lambda+3\mu)(\lambda+2\mu)\lambda_3^2\frac{\partial\Phi_9(x)}{\partial x_3}, \tag{2.9}$$

where

$$\Delta\Phi_9(x)=0,\quad \lambda_3^2=\frac{\eta_2(\lambda+2\mu)-\eta^2}{\eta_1(\lambda+2\mu)}>0.$$

Substituting the value of $\mathrm{div}u(x)$, from (2.9) into equation (2.6) and (2.8), we obtain

$$(\Delta-\lambda_3^2)v(x)=\frac{\eta\gamma-(\lambda+2\mu)\gamma_1}{\eta_1(\lambda+2\mu)}\left(2\frac{c}{|x|}+\frac{\partial\Phi_4(x)}{\partial x_3}\right)+$$
$$+\left(\frac{\eta\gamma-(\lambda+2\mu)\gamma_1}{\eta_1}\lambda_1^2-\frac{\varkappa\varkappa_7}{\varkappa_1\eta_1}(\lambda+2\mu)\lambda_1^4\right)\Phi_5(x)+\eta(\lambda+3\mu)\lambda_3^2\frac{\partial\Phi_9(x)}{\partial x_3}$$

From these, we get

$$\begin{aligned}2v(x)\ &=a_1\left(\frac{2c}{|x|}+\frac{\partial\Phi_4(x)}{\partial x_3}\right)+a_2\Phi_5(x)-\eta(\lambda+3\mu)\frac{\partial\Phi_9(x)}{\partial x_3}+\\&+(\lambda+2\mu)\lambda_3^2\Phi_8(x),\end{aligned} \tag{2.10}$$

where $(\Delta-\lambda_3^2)\Phi_8(x)=0$,

$$a_1\ =\frac{(\lambda+2\mu)\gamma_1-\gamma\eta}{\eta_1(\lambda+2\mu)\lambda_3^2},\quad a_2=\frac{\lambda_1^2(\lambda+2\mu)}{\eta_1(\lambda_3^2-\lambda_1^2)}\left(\frac{\gamma_1(\lambda+2\mu)-\gamma\eta}{\lambda+2\mu}+\frac{\varkappa\varkappa_7}{\varkappa_1}\lambda_1^2\right)$$

Substitute the expressions of $\theta(x)$ and $v(x)$, given by (2.6) and (2.10) into (2.1), to obtain

$$\begin{aligned}3\mu\Delta u(x)\ &+(\lambda+\mu)\mathrm{graddiv}u(x)=\mathrm{grad}[(\gamma-\eta a_1)\left(2\frac{c}{|x|}+\frac{\partial\Phi_4(x)}{\partial x_3}\right)+\\&+(\gamma\lambda_1^2(\lambda+2\mu)-\eta a_2)\Phi_5(x)+\eta^2(\lambda+3\mu)\frac{\partial\Phi_9(x)}{\partial x_3}-\\&-\eta(\lambda+2\mu)\lambda_3^2\Phi_8(x)],\end{aligned} \tag{2.11}$$

which implies

$$u(x)=u_0(x)+\tilde{u}(x), \tag{2.12}$$

(where $u_0(x)$ is a general solution of the Lame homogeneous equation

$$\mu\Delta u_0(x)+(\lambda+\mu)\mathrm{graddiv}u_0(x)=0,$$

and $\tilde{u}(x)$ is a particular solution of the nonhomogeneous system (2.11)

The solution $u_0(x)$ has the form [8]

$$u_0(x)=\mathrm{grad}\Phi_1(x)+ax_3\mathrm{grad}\Phi_2(x)-e_3\Phi_2(x)+\mathrm{rot}(e_3\Phi_3(x)),$$

where $a=(\lambda+\mu)/(\lambda+3\mu)$, $\Delta\Phi_j(x)=0$, $j=1,2,3$.

The particular solution of the system (2.11) will be written as

$$\begin{aligned}2\tilde{u}(x)\ &=(\gamma-\eta a_1)\left(\frac{c}{\lambda+2\mu}\frac{x}{|x|}+\frac{1}{\lambda+3\mu}x+x_3\mathrm{grad}\Phi_4(x)\right)+\\&+a_3\mathrm{grad}\Phi_5(x)-\eta^2x_3\mathrm{grad}\Phi_9(x)-\eta\mathrm{grad}\Phi_8(x),\end{aligned}$$

where

$$a_3=\gamma-\frac{\eta}{\lambda_1^2(\lambda+2\mu)}a_2.$$

Substituting the values of the vectors $u_0(x)$ and $\tilde{u}(x)$ into (2.15), we get

$$u(x) = \operatorname{grad}\Phi_1(x) + \alpha x_3 \operatorname{grad}\Phi_2(x) - e_3\Phi_2(x) + \operatorname{rot}(e_3\Phi_3(x)) + \\ +(\gamma - \eta\alpha_1)\left(\frac{c}{\lambda+2\mu}\frac{x}{|x|} + \frac{1}{\lambda+3\mu}x_3\operatorname{grad}\Phi_4(x)\right) + \\ +a_3\operatorname{grad}\Phi_5(x) + \eta^2 x_3 \operatorname{grad}\Phi_9(x) - \eta\operatorname{grad}\Phi_8(x). \tag{2.13}$$

Substitute the expressions of $u(x)$, $\theta(x)$ and $v(x)$, given by (2.13), (2.6) and (2.10) respectively, we obtain

$$(\eta^2 - (\lambda+3\mu)\eta_2)\frac{\partial\Phi_9(x)}{\partial x_3} - \frac{2\mu}{\lambda+3\mu}\frac{\partial\Phi_2(x)}{\partial x_3} - \frac{\mu\alpha_1}{\lambda+3\mu}\frac{\partial\Phi_4(x)}{\partial x_3} = 0.$$

This equality well be satisfied, if the function $\Phi_9(x)$ is defined in the following form

$$\Phi_9(x) = \frac{\mu}{(\lambda+3\mu)(\eta^2 - (\lambda+3\mu)\eta_2)}(2\Phi_2(x) + \alpha_1\Phi_4(x)),$$

where

$$\alpha_1 = \frac{\gamma\eta_2 - \eta\gamma_1}{\eta_2(\lambda+2\mu) - \eta^2}.$$

Substituting the value of $\Phi_9(x)$ into (2.10) and (2.13), we get

$$v(x) = 2a_1\frac{c}{|x|} + a_5\frac{\partial\Phi_2(x)}{\partial x_3} + a_4\frac{\partial\Phi_4(x)}{\partial x_3} + a_2\Phi_5(x) + (\lambda+2\mu)\lambda_3^2\Phi_8(x),$$

$$u(x) = \alpha_1 c\frac{x}{|x|} + \operatorname{grad}\Phi_1(x) + \alpha_2 x_3\operatorname{grad}\Phi_2(x) - e_3\Phi_2(x) + \\ +\operatorname{rot}(x\Phi_3(x)) - \alpha_3 x_3\operatorname{grad}\Phi_4(x) + a_3\operatorname{grad}\Phi_5(x) - \eta\operatorname{grad}\Phi_8(x),$$

where

$$a_4 = a_1 + \frac{\mu\eta\alpha_3}{\eta^2 - \eta_2(\lambda+2\mu)}, \quad a_5 = \frac{2\mu\eta}{\eta_2(\lambda+3\mu) - \eta^2}, \quad \alpha_3 = \frac{\eta_2\gamma - \eta\gamma_1}{\eta^2 - \eta_2(\lambda+3\mu)}.$$

Thereby we have proved the first part of the theorem. As to the second part, it is proved by a straightforward verification that the vector $U = (u, w, \theta, v)^\top$ represented in form (2.5) is a solution of system (2.1) – (2.4). □

Denote by Ω^- a half-space $x_3 > 0$, and by $\partial\Omega$ its boundary plane $x_3 = 0$, $\Omega_R := \Omega^- \cap B(O,R)$, where $B(O,R)$ is the ball with center at the origin and radius R. Denote by $\sum_R$ that part of the boundary of the ball $B(O,R)$ which lies in the domain $x_3 > 0$, by $S(O,R)$ the circle with center at the origin and radius R which lies on the plane $x_3 = 0$.

Definition. Assume that in the domain Ω^-, the regular vector $U = (u, w, \theta)^\top$ has the property $Z(\Omega^-)$ if it satisfies the following conditions

$$u(x) = O(1), \quad \theta(x) = O(|x|^{-1}), \quad v(x) = O(|x|^{-1}), \quad w(x) = O(|x|^{-2}), \tag{2.14}$$

$$\lim_{R\to\infty}\frac{1}{2\pi R^2}\int_{\Sigma_R} n(x)\cdot u(x)d\Sigma_R = 0, \tag{2.15}$$

where $n(x)$ is the external normal unit vector passing at a point $x \in \Sigma_R$ with respect to Ω_R, $R = |x|$

Theorem 2. *A solution of system (2.1) –(2.4) which has the property* $Z(\Omega^-)$

$$\begin{aligned} u(x) &= \operatorname{grad}\Phi_1(x) + \alpha_2 x_3\operatorname{grad}\Phi_2(x) - e_3\Phi_2(x) + \operatorname{rot}\big(e_3\Phi_3(x)\big) - \\ &\quad -\alpha_3 x_3\operatorname{grad}\Phi_4(x) + a_3\operatorname{grad}\Phi_5(x) - \eta\operatorname{grad}\Phi_8(x), \\ w(x) &= \operatorname{grad}\left(\alpha_4\frac{\partial\Phi_4(x)}{\partial x_3} + \alpha_5\Phi_5(x)\right) + \operatorname{rotrot}\big(e_3\Phi_6(x)\big) + \operatorname{rot}\big(e_3\Phi_7(x)\big), \\ \theta(x) &= \frac{\partial\Phi_4(x)}{\partial x_3} + (\lambda+2\mu)\lambda_1^2\Phi_5(x), \\ v(x) &= a_5\frac{\partial\Phi_2(x)}{\partial x_3} + a_4\frac{\partial\Phi_4(x)}{\partial x_3} + a_2\Phi_5(x) + (\lambda+2\mu)\lambda_3^2\Phi_8(x). \end{aligned} \tag{2.16}$$

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Proof. The proof of this theorem follows from Theorem 2.1 the vectors $u(x)$ and $w(x)$ and the functions $\theta(x)$ and $v(x)$ represented by formulas (2.5) satisfy conditions (2.14). If the value of $u(x)$ is substituted from (2.5) into (2.15), then we have $c = 0$. If the value $c = 0$ is used in (2.5), then we obtain equalities (2.16), which completes the proof of the theorem. □

Remark 2.3. The solution of differential equations (2.1) –(2.4), with have $Z(\Omega^-)$ property, a point at infinity satisfies the following conditions under in vanishes

$$\begin{gathered} u(x) = O(\ |x|^{-1}),\ \ \theta(x) = O(\ |x|^{-2}),\ \ v(x) = O(\ |x|^{-2}),\ \ w(x) = O(\ |x|^{-3}), \\ \partial_k u(x) = O(\ |x|^{-2}),\quad k = 1,2,3, \end{gathered} \tag{2.17}$$

where $\partial_k = \partial/\partial x_k,\quad k = 1,2,3.$

3. Statement of the problem. The uniqueness theorem

Problem $(\mathbf{III})^-$. Find, in the domain Ω^-, a vector $U = (u, w, \theta, v)^\top$ with property $Z(\Omega^-)$ that in this domain satisfies system (2.1) –(2.4) and on the boundary $\partial\Omega$, the following boundary conditions:

$$\begin{gathered} \{n(y)\cdot u(y)\}^- = f'_3(y),\quad \{P^{(1)}(\partial,n)U'(y) - n(n\cdot P^{(1)}(\partial,n)U'(y))\}^- = f'(y), \\ \{n(y)\cdot w(y)\}^- = f''_3(y),\quad \{P^{(2)}(\partial,n)U''(y) - n(n\cdot P^{(2)}(\partial,n)U''(y))\}^- = f''(y), \\ \{P^{(3)}(\partial,n)U''(y)\}^- = f'_4(y),\quad \{P^{(4)}(\partial,n)U'''(y)\}^- = f''_4(y), \end{gathered} \tag{3.1}$$

where $f' = (f'_1, f'_2, 0)^\top$, $f'' = (f''_1, f''_2, 0)^\top$,$f'_j,\ f''_j,\ \ j = 1,2,3,4$ are the functions given on the boundary $\partial\Omega$, $n(y)$ is the external normal unit vector passing at a point $y \in \partial\Omega$, i.e. $n(y) = (o, o, 1)^\top$

Theorem 4. *If Problem* $(\mathbf{III})^-$ *have solution, these solution is unique.*

Proof. The theorem will be proved if we show that the homogeneous $(f'_j = 0,\ \ f''_j = 0,\ \ j = 1,2,3,4)$ Problem $(\mathbf{III})_0^-$ have only trivial solution .

Assume that the vector $U = (u, w, \theta, v)^\top$ is a solution of Problem $(\mathbf{III})_0^-$.If we multiply both sides of equality (2.1)–(2.4). by the u, w, θ and v, respectively and integrate over the domain Ω_R, then, using Stoke's formula and boundary conditions

$$\begin{gathered} \{u(y)\cdot P^{(1)}(\partial,n)U'(y)\}^- = 0,\quad \{w(y)\cdot P^{(2)}(\partial,n)U''(y)\}^- = 0, \\ \{\theta(y)P^{(3)}(\partial,n)U''(y)\}^- = 0,\quad \{v(x)\cdot P^{(4)}(\partial,n)U'''(y)\}^- = 0, \end{gathered}$$

we get

$$\int_{\Sigma_R} u\,(x)\cdot P^{(1)}(\partial,n)U'(x)d\Sigma_R - \int_{\Omega_R}[\,E^{(1)}(u,u) - \gamma\theta(x)\mathrm{div}u(x) + \eta v(x)\mathrm{div}u(x)]dx = 0, \tag{3.2}$$

$$\int\limits_{\Sigma_R} w\,(x)\cdot P^{(2)}(\partial,n)U''(x)d\Sigma_R -$$

$$- \int_{\Omega_R}[\,E^{(2)}(w,w) + \varkappa_3 w(x)\cdot\mathrm{grad}\theta(x) + \varkappa_2 w^2(x)]dx = 0, \tag{3.3}$$

$$\int_{\Sigma_R}\theta\,(x)\cdot P^{(3)}(\partial,n)U''(x)d\Sigma_R - \int_{\Omega_R}[\,\varkappa\mathrm{grad}^2\theta(x) + \varkappa_1 w(x)\cdot\mathrm{grad}\theta(x)]dx = 0, \tag{3.4}$$

$$\int\limits_{\Sigma_R} v\,(x)\cdot P^{(4)}(\partial,n)U'''(x)d\Sigma_R - \int\limits_{\Omega_R}[\eta_1\mathrm{grad}^2 v(x) + \eta v(x)\mathrm{div}u(x) +$$

$$+\eta_2 v^2(x) - \gamma_1 v(x)\theta(x) - \varkappa_7 w(x)\cdot\mathrm{grad}v(x)]dx, \tag{3.5}$$

where

$$E^{(1)}(u,u) = \frac{3\lambda+2\mu}{3}(\mathrm{div}u)^2 + \frac{\mu}{2}\sum_{k\neq j=1}^{3}\left(\frac{\partial u_k}{\partial x_j}+\frac{\partial u_j}{\partial x_k}\right)^2 + \frac{\mu}{3}\sum_{k,j=1}^{3}\left(\frac{\partial u_k}{\partial x_j}-\frac{\partial u_j}{\partial x_k}\right)^2$$

$$E^{(2)}(w,w) = \frac{3\varkappa_4+\varkappa_5+\varkappa_6}{3}(\mathrm{div}w)^2 + \frac{\varkappa_6-\varkappa_5}{2}(\mathrm{rot}w)^2 + \\ + \frac{\varkappa_5+\varkappa_6}{4}\sum_{k\neq j=1}^{3}\left(\frac{\partial w_k}{\partial x_j}-\frac{\partial w_j}{\partial x_k}\right)^2 + \frac{\varkappa_5+\varkappa_6}{6}\sum_{k,j=1}^{3}\left(\frac{\partial w_k}{\partial x_k}-\frac{\partial w_j}{\partial x_j}\right)^2.$$

Passing to the limit on both sides of equalities (3.2) –(3.5) as $R\to+\infty$ and taking into consideration the asymptotic representations (2.17), we obtain

$$\int_{\Omega^-}[E^{(1)}(u,u)+\gamma\theta(x)\mathrm{div}u(x)+\eta v(x)\mathrm{div}u(x)]dx=0, \quad (3.6)$$

$$\int_{\Omega^-}[E^{(2)}(w,w)+\varkappa_3 w(x)\cdot\mathrm{grad}\theta(x)+\varkappa_2 w^2(x)]dx=0, \quad (3.7)$$

$$\int_{\Omega^-}[\varkappa\,\mathrm{grad}^2\theta(x)+\varkappa_1 w(x)\cdot\mathrm{grad}\theta(x)]dx=0, \quad (3.8)$$

$$\int_{\Omega^-}[\eta_1\mathrm{grad}^2 v(x)+\eta v(x)\mathrm{div}u(x)+\eta_2 v^2(x)- \\ -\gamma_1 v(x)\theta(x)-\varkappa_7 w(x)\cdot\mathrm{grad}v(x)]dx=0. \quad (3.9)$$

From the equalities (3.7)- (3.8), we get

$$\int_{\Omega^-}\Big\{T_0E^{(2)}(w,w)+\frac{4T_0\varkappa\varkappa_0-(\varkappa_1+T_0\varkappa_3)^2}{4\varkappa}+\frac{1}{4\varkappa}[(\varkappa_1+T_0\varkappa_3)w(x)+ \\ +2\varkappa\,\mathrm{grad}\theta(x)]^2\Big\}dx=0. \quad (3.10)$$

Since $E^{(2)}(w,w)\geq 0$, $4T_0\varkappa\varkappa_2>(\varkappa_1+T_0\varkappa_3)^2$, $\varkappa>0$, $T_0>0$, form (3.10) we obtained that $w(x)=0$, $\theta(x)=c=const$, $x\in\Omega^-$. By asymptotic (2.17) we have $c=0$, i.e. $\theta(x)=0$, $x\in\Omega^-$.

Substituting the value of the functions $w(x)=0$, $\theta(x)=0$, $x\in\Omega^-$, into (3.6) and (3.9), we obtain

$$\int_{\Omega^-}[E^{(1)}(u,u)+2\eta v(x)\mathrm{div}u(x)+\eta_1\mathrm{grad}^2 v(x)+\eta_2 v^2(x)]dx=0 \quad (3.11)$$

$$E(\widetilde{U},\widetilde{U})=E^{(1)}(u,u)+2\eta v(x)\mathrm{div}u(x)+\eta_1\mathrm{grad}^2v(x)+\eta_2v^2(x)= \\ =\frac{\mu}{2}\sum_{k\neq j=1}^{3}\left(\frac{\partial u_k}{\partial x_j}+\frac{\partial u_j}{\partial x_k}\right)^2+\frac{\mu}{3}\sum_{k,j=1}^{3}\left(\frac{\partial u_k}{\partial x_k}-\frac{\partial u_j}{\partial x_j}\right)^2+\eta_1\mathrm{grad}^2v(x)+ \\ +\frac{(3\lambda+2\mu)\eta_2-3\eta^2}{3\lambda+2\mu}v^2(x)+\frac{1}{3(3\lambda+2\mu)}(3\eta v(x)+(3\lambda+2\mu)\mathrm{div}v(x))^2, \quad (3.12)$$

where $\widetilde{U}=(u,v)^{\top}$.

Tacking into account that $3\lambda+2\mu>0$, $\eta_1>0$, $\mu>0$, $(3\lambda+2\mu)\eta_2-3\eta^2>0$, from (3.12) it follows that $E(\widetilde{U},\widetilde{U})\geq 0$, By virtue of this fact, (3.11) implies

$$E(\widetilde{U},\widetilde{U})=0, \quad x\in\Omega^-.$$

A solution of this equations has the form

$$u(x)=[b\times x]+d, \quad v(x)=0, \quad x\in\Omega^-,$$

where b u d are any three-dimensional constant vectors. By asymptotic (2.17) we have $b = d = 0$, i.e. $u(x) = 0, \quad x \in \Omega^-$. □

4.Solution of the boundary value problems

If in the boundary conditions (3.1) we assume that $n(y) = (0,0,1)^\top$, then these boundary conditions can be rewritten as follows:

$$\begin{aligned}
&\{u_3(y)\}^- = f'_3(y), \quad \{\mathrm{rot}u(y)\}_j^- = F_j'(y), \quad j = 1,2,\\
&\{w_3(y)\}^- = f''_3(y), \quad \{\mathrm{rot}w(y)\}_j^- = F_j''(y), \quad j = 1,2,\\
&\left\{\varkappa \frac{\partial\theta(y)}{\partial x_3} + \varkappa_1 w_3(y)\right\}^- = f'_4(y), \quad \left\{\eta \frac{\partial v(y)}{\partial x_3} + \varkappa_7 w_3(y)\right\}^- = f''_4(y),
\end{aligned} \tag{4.1}$$

where

$$F'_j(y) = \frac{1}{\mu}\left(\delta_{2j} f_1'(y) - \delta_{1j} f_2'(y)\right) + 2\left(\delta_{1j}\frac{\partial}{\partial y_2} - \delta_{2j}\frac{\partial}{\partial y_1}\right) f'_3(y),$$

$$F''_j(y) = \frac{1}{\varkappa_6}\left(\delta_{2j} f_1''(y) - \delta_{1j} f_2''(y)\right) + \frac{\varkappa_5 + \varkappa_6}{\varkappa_6}\left(\delta_{1j}\frac{\partial}{\partial y_2} - \delta_{2j}\frac{\partial}{\partial y_1}\right) f''_3(y).$$

In formulas (4.1) assume the following

$$\left\{\frac{\partial\varphi(y)}{\partial x_3}\right\}^- = \lim_{\Omega^- \ni x \to y \in \partial\Omega} \frac{\partial\varphi(y)}{\partial x_3}.$$

A solution of this problem $(\mathbf{III})_0^-$ is sought in form (2.16). Functions $\Phi_j(x)$, $j = 1,2,\ldots,8$ are represented as follows

$$\begin{aligned}
\Phi_j(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty} e^{-i(\tilde{x}\cdot\xi) - x_3|\xi|}\, g_j(\xi) d\xi_1 d\xi_2, \quad j = 1,2,3,4,\\
\Phi_5(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty} e^{-i(\tilde{x}\cdot\xi) - x_3\sqrt{|\xi|^2 + \lambda_1^2}}\, g_5(\xi) d\xi_1 d\xi_2,\\
\Phi_j(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty} e^{-i(\tilde{x}\cdot\xi) - x_3\sqrt{|\xi|^2 + \lambda_2^2}}\, g_j(\xi) d\xi_1 d\xi_2, \quad j = 6,7,\\
\Phi_8(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty} e^{-i(\tilde{x}\cdot\xi) - x_3\sqrt{|\xi|^2 + \lambda_3^2}}\, g_8(\xi) d\xi_1 d\xi_2,
\end{aligned} \tag{4.2}$$

where $g_j(\xi), \quad j = 1,2,\ldots,8$ are the sought functions, $\xi = (\xi_1, \xi_2)^\top$, $\tilde{x} = (x_1, x_2)^\top$, $x = (x_1, x_2, x_3)$, $|\xi| = \sqrt{\xi_1^2 + \xi_2^2}$.

If the functions $\Phi_j(x), \quad j = 1,2,\ldots,8$ defined by (4.2) is substituted into (2.16), we obtain

$$
\begin{aligned}
u(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\{[-e_1g_1(\xi)-(\alpha_2x_3e_1+e_3)g_2(\xi)+e_2g_3(\xi)\\
&+\alpha_3x_3e_1g_4(\xi)]e^{-x_3|\xi|}-a_3e_4g_5(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_1^2}}+\\
&+\eta e_6g_8(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_3^2}}\Big\}e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,\\
w(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\Big\{\alpha_4|\xi|e_1g_4(\xi)e^{-x_3|\xi|}-\alpha_5e_4g_5(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_1^2}}+\\
&+\Big[\Big(e_5\sqrt{|\xi|^2+\lambda_2^2}-\lambda_2^2e_3\Big)g_6(\xi)+e_2g_7(\xi)\Big]e^{-x_3\sqrt{|\xi|^2+\lambda_2^2}}\Big\}e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,\\
\theta(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\Big[-|\xi|g_4(\xi)e^{-x_3|\xi|}+(\lambda+2\mu)\lambda_1^2g_5(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_1^2}}\Big]e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,\\
v(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\Big[-|\xi|\big(a_5g_2(\xi)+a_4g_4(\xi)\big)e^{-x_3|\xi|}+a_2g_5(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_1^2}}+\\
&+(\lambda+2\mu)\lambda_3^2g_8(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_3^2}}\Big]e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,
\end{aligned}
\tag{4.3}
$$

where $e_1=(i\xi_1,i\xi_2,|\xi|)^\top$, $e_2=(-i\xi_2,i\xi_1,0)^\top$, $e_3=(0,0,1)^\top$, $e_4=(i\xi_1,i\xi_2,\sqrt{|\xi|^2+\lambda_1^2})^\top$, $e_5=(i\xi_1,i\xi_2,\sqrt{|\xi|^2+\lambda_2^2})^\top$, $e_6=(i\xi_1,i\xi_2,\sqrt{|\xi|^2+\lambda_3^2})^\top$.

From (4.3) we obtain

$$
\begin{aligned}
\{\mathrm{rot}u(x)\}_j &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\big[(\alpha_2+1)(i\xi_2\delta_{1j}-i\xi_1\delta_{2j})g_2(\xi)+i\xi_j|\xi|g_3(\xi)-\\
&-\alpha_3(i\xi_2\delta_{1j}-i\xi_1\delta_{2j}g_2(\xi))g_4(\xi)\big]e^{-i(\tilde{x}\cdot\xi)-x_3|\xi|}d\xi_1d\xi_2,\quad j=1,2,\\
\{\mathrm{rot}w(x)\}_j &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\big[\lambda_2^2(i\xi_2\delta_{1j}-i\xi_1\delta_{2j})g_6(\xi)+\\
&+i\xi_j\sqrt{|\xi|^2+\lambda_2^2}g_7(\xi)\big]e^{-i(\tilde{x}\cdot\xi)-x_3\sqrt{|\xi|^2+\lambda_2^2}}d\xi_1d\xi_2,\quad j=1,2,\\
\varkappa\frac{\partial\theta(x)}{\partial x_3}+\varkappa_1w_3(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\big[\alpha_6|\xi|^2g_4(\xi)e^{-x_3|\xi|}+\\
&+\varkappa_1|\xi|^2g_6(\xi)e^{-x_3\sqrt{|\xi|^2+\lambda_2^2}}\Big]e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,\\
\eta_1\frac{\partial v(x)}{\partial x_3}-\varkappa_7w_3(x) &= \frac{1}{2\pi}\iint_{-\infty}^{+\infty}\big[(\eta_1a_5|\xi|^2g_2(\xi)+\alpha_7|\xi|^2g_4(\xi))e^{-x_3|\xi|}-\\
&-\alpha_8\sqrt{|\xi|^2+\lambda_1^2}e^{-x_3\sqrt{|\xi|^2+\lambda_1^2}}g_5(\xi)-\varkappa_7|\xi|^2e^{-x_3\sqrt{|\xi|^2+\lambda_2^2}}g_6(\xi)-\\
&-\eta_1(\lambda+2\mu)\lambda_3^2\sqrt{|\xi|^2+\lambda_3^2}e^{-x_3\sqrt{|\xi|^2+\lambda_3^2}}g_8(\xi)\Big]e^{-i(\tilde{x}\cdot\xi)}d\xi_1d\xi_2,
\end{aligned}
\tag{4.4}
$$

where

$$\alpha_6=\alpha_4\varkappa_1+\varkappa,\quad \alpha_7=\eta_1a_4-\varkappa_7\alpha_4,\quad \alpha_8=a_2\eta_1-\alpha_5\varkappa_7.$$

From (4.3)-(4.4) with boundary conditions (4.1) taken into account we obtain

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$$\begin{aligned}
&\frac{1}{2\pi}\iint\limits_{-\infty}^{+\infty}\left[-|\xi|g_1(\xi)-g_2(\xi)-\alpha_3\sqrt{|\xi|^2+\lambda_1^2}g_4(\xi)+\right.\\
&\qquad\left.+\eta\sqrt{|\xi|^2+\lambda_3^2}g_8(\xi)\right]e^{-i(y\cdot\xi)}d\xi_1d\xi_2=f_3{}'(y),\\
&\frac{1}{2\pi}\iint\limits_{-\infty}^{+\infty}\left[\alpha_4|\xi|^2g_4(\xi)-\alpha_5\sqrt{|\xi|^2+\lambda_1^2}g_5(\xi)+|\xi|^2g_6(\xi)\right]e^{-i(y\cdot\xi)}d\xi_1d\xi_2=f_3{}''(y),\\
&\frac{1}{2\pi}\iint_{-\infty}^{+\infty}[\alpha_6|\xi|^2g_4(\xi)+\varkappa_1|\xi|^2g_6(\xi)]\,e^{-i(y\cdot\xi)}d\xi_1d\xi_2=f_4{}'(y),\\
&\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\left[\eta_1a_5|\xi|^2g_2(\xi)+\alpha_7|\xi|^2g_4(\xi)-\alpha_8\sqrt{|\xi|^2+\lambda_1^2}g_5(\xi)-\right.\\
&\qquad\left.-\varkappa_7|\xi|^2g_6(\xi)-\eta_1(\lambda+2\mu)\lambda_3^2\sqrt{|\xi|^2+\lambda_3^2}g_8(\xi)\right]e^{-i(y\cdot\xi)}d\xi_1d\xi_2=f_4{}''(y),\\
&\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\big[(\alpha_2+1)(i\xi_2\delta_{1j}-i\xi_1\delta_{2j})g_2(\xi)+i|\xi|\xi_jg_3(\xi)-\\
&\qquad-\alpha_3(i\xi_2\delta_{1j}-i\xi_1\delta_{2j})g_4(\xi)\big]e^{-i(y\cdot\xi)}d\xi_1d\xi_2=F_j{}'(y),\quad j=1,2,\\
&\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\left[\lambda_2^2(i\xi_2\delta_{1j}-i\xi_1\delta_{2j})g_6(\xi)+i\xi_j\sqrt{|\xi|^2+\lambda_2^2}g_7(\xi)\right]e^{-i(y\cdot\xi)}d\xi_1d\xi_2=\\
&\qquad=F_j{}''(y),\quad j=1,2,
\end{aligned}\tag{4.5}$$

From equalities (4.5) we have

$$\begin{aligned}
&i\xi_2(\alpha_2+1)g_2(\xi)+i\xi_1|\xi|g_3(\xi)-i\xi_2\alpha_3g_4(\xi)=\hat{F}_1{}'(\xi),\\
-\;&i\xi_1(\alpha_2+1)g_2(\xi)+i\xi_2|\xi|g_3(\xi)+i\xi_1\alpha_3g_4(\xi)=\hat{F}_2{}'(\xi),\\
&i\xi_2\lambda_2^2g_6(\xi)+i\xi_1\sqrt{|\xi|^2+\lambda_2^2}\;g_7(\xi)=\hat{F}_1{}''(\xi),\\
-\;&i\xi_1\lambda_2^2g_6(\xi)+i\xi_2\sqrt{|\xi|^2+\lambda_2^2}\;g_7(\xi)=\hat{F}_2{}''(\xi),\\
-\;&|\xi|g_1(\xi)-g_2(\xi)-a_3\sqrt{|\xi|^2+\lambda_1^2}\;g_5(\xi)+\eta\sqrt{|\xi|^2+\lambda_3^2}\;g_8(\xi)=\hat{f}_3'(\xi),\\
&\alpha_4|\xi|^2g_4(\xi)-\alpha_5\sqrt{|\xi|^2+\lambda_1^2}\;g_5(\xi)+|\xi|^2\;g_6(\xi)=\hat{f}_3{}''(\xi),\\
&\alpha_6|\xi|^2g_4(\xi)+\varkappa_1|\xi|^2g_6(\xi)=\hat{f}_4{}'(\xi),\\
&\eta_1a_5|\xi|^2g_2(\xi)+\alpha_7|\xi|^2g_4(\xi)-\alpha_8\sqrt{|\xi|^2+\lambda_1^2}\;g_5(\xi)-\varkappa_7|\xi|^2g_6(\xi)-\\
&-\eta_1(\lambda+2\mu)\lambda_3^2\sqrt{|\xi|^2+\lambda_3^2}\;g_8(\xi)=\hat{f}_4{}''(\xi),
\end{aligned}\tag{4.6}$$

where

$$\begin{aligned}
&\hat{F}_j'(\xi)=\frac{1}{\mu}\Big(\delta_{2j}\hat{f}_1'(\xi)-\delta_{1j}\hat{f}_2'(\xi)\Big)+2\big(i\xi_1\delta_{2j}-i\xi_2\delta_{1j}\big)\hat{f}_3'(\xi),\quad j=1,2,\\
&\hat{F}''(\xi)=\frac{1}{\varkappa_6}\Big(\delta_{2j}\hat{f}_1''(\xi)-\delta_{1j}\hat{f}_2''(\xi)\Big)+\frac{\varkappa_5+\varkappa_6}{\varkappa_6}\big(i\xi_1\delta_{2j}-i\xi_2\delta_{1j}\big)\hat{f}_3''(\xi),\quad j=1,2,\\
&\hat{f}_j'(\xi)=\frac{1}{2\pi}\iint_{-\infty}^{+\infty}e^{i(y\cdot\xi)}\,f_j'(y)dy_1dy_2,\quad j=1,2,3,4,\\
&\hat{f}_j''(\xi)=\frac{1}{2\pi}\iint_{-\infty}^{+\infty}e^{i(y\cdot\xi)}\,f_j''(y)dy_1dy_2,\quad j=1,2,3,4.
\end{aligned}\tag{4.7}$$

A solution of system (4.6) has the form

$$
\begin{aligned}
g_1(\xi) &= \tfrac{\alpha_{12}}{|\xi|^3}\left(|\xi|^2 - \lambda_6^2\right)\left(\tilde{e}_1 \cdot \hat{f}'(\xi)\right) - \tfrac{\alpha_{13}}{|\xi|^3}\left(|\xi|^2 - \lambda_7^2\right)\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) - \\
&\quad - \tfrac{\alpha_{14}}{|\xi|}\left(|\xi|^2 + \lambda_8^2\right)\hat{f}_3'(\xi) + \tfrac{\alpha_{15}}{|\xi|}\left(|\xi|^2 + \lambda_9^2\right)\hat{f}_3''(\xi) - \\
&\quad - \tfrac{\alpha_{16}}{|\xi|^3}\left(|\xi|^2 + \lambda_{10}^2\right)\hat{f}_4'(\xi) - \tfrac{\eta}{\eta_1(\lambda+2\mu)\lambda_3^2|\xi|}\hat{f}_4''(\xi) \\
g_2(\xi) &= \tfrac{1}{(\alpha_2+1)|\xi|^2}\left[\tfrac{1}{\mu}\left(\tilde{e}_1 \cdot \hat{f}'(\xi)\right) - \tfrac{\varkappa_1\alpha_3}{\alpha_6\varkappa_2}\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) + \right. \\
&\quad \left. + \tfrac{\alpha_3}{\alpha_6}\hat{f}_4'(\xi)\right] + \tfrac{1}{\alpha_2+1}\left[\tfrac{\alpha_3\varkappa_1}{\varkappa\lambda_4^2}\hat{f}_3''(\xi) - 2\hat{f}_3'(\xi)\right], \\
g_3(\xi) &= \tfrac{1}{\mu|\xi|^3}\left(\tilde{e}_2 \cdot \hat{f}'(\xi)\right), \\
g_4(\xi) &= -\tfrac{\varkappa_1}{\alpha_6\varkappa_2|\xi|^2}\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) + \tfrac{\varkappa_1}{\varkappa\lambda_4^2}\hat{f}_3''(\xi) + \tfrac{1}{\alpha_6|\xi|^2}\hat{f}_4'(\xi), \\
g_5(\xi) &= \tfrac{1}{\alpha_5\sqrt{|\xi|^2+\lambda_1^2}}\left[\tfrac{\varkappa}{\varkappa_2\alpha_6}\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) - \tfrac{1}{\lambda_4^2}\left(|\xi|^2 + \lambda_4^2\right)\hat{f}_3''(\xi) + \right. \\
&\quad \left. + \tfrac{\alpha_4}{\alpha_6}\hat{f}_4'(\xi)\right], \\
g_6(\xi) &= \tfrac{1}{\varkappa_2|\xi|^2}\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) - \tfrac{\alpha_6}{\varkappa\lambda_4^2}\hat{f}_3''(\xi), \\
g_7(\xi) &= \tfrac{1}{\varkappa_6|\xi|^2\sqrt{|\xi|^2+\lambda_2^2}}\left(\tilde{e}_2 \cdot \hat{f}''(\xi)\right), \\
g_8(\xi) &= \tfrac{1}{(\lambda+2\mu)\lambda_3^2\sqrt{|\xi|^2+\lambda_3^2}}\left[\tfrac{a_5}{\mu(\alpha_2+1)}\left(\tilde{e}_1 \cdot \hat{f}'(\xi)\right) - \alpha_9\left(\tilde{e}_1 \cdot \hat{f}''(\xi)\right) - \right. \\
&\quad \left. - \tfrac{2a_5}{\alpha_2+1}|\xi|^2\hat{f}'(\xi) + \alpha_{11}(|\xi|^2 + \lambda_5^2)\hat{f}_3{}''(\xi) + \alpha_{10}\hat{f}_4{}'(\xi) - \tfrac{1}{\eta_1}\hat{f}_4{}''(\xi)\right]
\end{aligned}
\tag{4.8}
$$

where $\tilde{e}_1 = (i\xi_1,\ i\xi_2)^\top$, $\tilde{e}_2 = (-i\xi_2,\ i\xi_1)^\top$, $\hat{f}' = (\hat{f}'_1,\ \hat{f}'_2)^\top$, $\hat{f}'' = (\hat{f}''_1,\ \hat{f}''_2)^\top$

$$
\lambda_4^2 = \frac{\varkappa_2\alpha_6}{\varkappa(\varkappa_5+\varkappa_6)}, \quad \lambda_5^2 = \frac{\varkappa\lambda_4^2(a_2\eta_1 - \alpha_2\varkappa_7)}{\eta_1(\varkappa_1 a_1\alpha_5 + \alpha_2\varkappa)}, \quad \lambda_6^2 = \frac{(\lambda+2\mu)\lambda_3^2}{\eta a_5},
$$

$$
\lambda_7^2 = \frac{\varkappa_1\alpha_3}{(\alpha_2+1)\varkappa_2\alpha_6\alpha_{13}}, \ \ \lambda_8^2 = \frac{\alpha_2-1}{(\alpha_2+1)\alpha_{14}}, \ \ \lambda_9^2 = \frac{1}{\alpha_{15}}\left(\frac{a_3}{\alpha_5} - \frac{\alpha_3\varkappa_1}{(\alpha_2+1)\varkappa\lambda_4^2} + \frac{\alpha_{11}\eta\lambda_5^2}{(\lambda+2\mu)\lambda_3^2}\right),
$$

$$
\alpha_9 = \frac{a_4\varkappa_1\alpha_5 + a_2\varkappa}{\varkappa_2\alpha_5\alpha_6}, \quad \alpha_{10} = \frac{a_1\alpha_5 - a_2\alpha_4}{\alpha_5\alpha_6}, \quad \alpha_{11} = \frac{a_1\varkappa_1\alpha_5 + a_2\varkappa}{\varkappa\alpha_5\lambda_4^2},
$$

$$
\alpha_{12} = \frac{\eta a_5}{\mu(\alpha_2+1)(\lambda+2\mu)\lambda_3^2}, \ \ \alpha_{13} = \frac{a_3\varkappa}{\varkappa_2\alpha_5\alpha_6} + \frac{\eta\alpha_9}{(\lambda+2\mu)\lambda_3^2}, \ \ \alpha_{14} = \frac{2\eta a_5}{(\alpha_2+1)(\lambda+2\mu)\lambda_3^2},
$$

$$
\alpha_{15} = \frac{a_3}{\alpha_5\lambda_4^2} + \frac{\alpha_{11}\eta}{(\lambda+2\mu)\lambda_3^2}, \ \ \alpha_{16} = \frac{a_3\alpha_4}{\alpha_5\alpha_6} - \frac{\eta\alpha_{10}}{(\lambda+2\mu)\lambda_3^2}, \ \ \lambda_{10}^2 = \frac{\alpha_3}{\alpha_6(\alpha_2+1)\alpha_{16}}.
$$

Substituting the values of the functions $g_j(\xi)$, $j = 1,2,\ldots,8$, from (4.8) into (4.3) and taking into account the following equalities (4.7) and

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$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{1}{|\xi|}e^{-i(\tilde{x}-y)\cdot\xi-x_3|\xi|}d\xi_1 d\xi_2=\frac{1}{r}$$

$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{\xi_k\xi_j}{|\xi|^3}e^{-i(\tilde{x}-y)\cdot\xi-x_3|\xi|}d\xi_1 d\xi_2=\frac{\partial^2\Phi(x,y)}{\partial x_k\partial x_j},\quad k,j=1,2,$$

$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{i\xi_k}{|\xi|^2}e^{-i(\tilde{x}-y)\cdot\xi-x_3|\xi|}d\xi_1 d\xi_2=\frac{\partial^2\Phi(x,y)}{\partial x_k\partial x_3},\quad k=1,2,$$

$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{1}{|\xi|^2}e^{-i(\tilde{x}-y)\cdot\xi-x_3|\xi|}d\xi_1 d\xi_2=-\frac{\partial\Phi(x,y)}{\partial x_3},$$

$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{i\xi_k}{|\xi|^3}e^{-i(\tilde{x}-y)\cdot\xi-x_3|\xi|}d\xi_1 d\xi_2=-\frac{\partial\Phi(x,y)}{\partial x_k},\quad k=1,2,$$

$$\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{1}{\sqrt{|\xi|^2+\lambda_j^2}}e^{-i(\tilde{x}-y)\cdot\xi-x_3\sqrt{|\xi|^2+\lambda_j^2}}d\xi_1 d\xi_2=\frac{e^{-\lambda_j r}}{r},\quad j=1,2,$$

where $r=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2+x_3^2}$, $\Phi(x,y)=x_3\ln(r+x_3)-r$, we obtain

$$U(x)=\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\mathbf{K}(x,y)f(y)dy_1dy_2, \tag{4.9}$$

where $=(u,w,\theta,v)^\top$ $f=(f_1{}',f_2{}',f_3{}',f_1{}'',f_2{}'',f_3{}'',f_4{}',f_4{}'')^\top$.

$$\mathbf{K}(x,y)=\begin{bmatrix}\mathbf{K}^{(1)}(x,y) & \mathbf{K}^{(2)}(x,y) & \mathbf{K}^{(5)}(x,y) & \mathbf{K}^{(6)}(x,y)\\ [0]_{3\times3} & \mathbf{K}^{(4)}(x,y) & \mathbf{K}^{(7)}(x,y) & [0]_{3\times1}\\ [0]_{1\times3} & \mathbf{K}^{(10)}(x,y) & \mathbf{K}^{(13)}(x,y) & 0\\ \mathbf{K}^{(11)}(x,y) & \mathbf{K}^{(12)}(x,y) & \mathbf{K}^{(15)}(x,y) & \mathbf{K}^{(16)}(x,y)\end{bmatrix}_{8\times8}$$

$\mathbf{K}^{(p)}(x,y)=[\mathbf{K}_{lj}^{(p)}(x,y)]_{3\times3}$, $p=1,2,4$, $\mathbf{K}^{(p)}(x,y)=[\mathbf{K}_{lj}^{(p)}(x,y)]_{3\times1}$, $p=5,6,7$, $\mathbf{K}^{(p)}(x,y)=[\mathbf{K}_{lj}^{(p)}(x,y)]_{1\times3}$, $p=10,11,12$, and $\mathbf{K}^{(p)}(x,y)$, $p=13,15,16$ are scalar functions,

$$\begin{aligned}\mathbf{K}_{lj}^{(1)}(x,y)\quad &=(1-\delta_{l3})(1-\delta_{3j})\left[-\frac{1}{\mu}\delta_{lj}\frac{1}{r}+\alpha_{12}\frac{\partial^2}{\partial x_l\partial x_j}\frac{e^{-\lambda_3 r}-1}{r}+\frac{\alpha_2}{\mu(\alpha_2+1)}\frac{\partial^2 r}{\partial x_l\partial x_j}\right]+\\ &+\delta_{3j}(1-\delta_{l3})\left[\frac{\mu}{\lambda+2\mu}\frac{\partial}{\partial x_l}\frac{1}{r}-\alpha_{14}\lambda_3^2\frac{\partial}{\partial x_l}\frac{e^{-\lambda_3 r}-1}{r}+\right.\\ &\left.+\alpha_{14}\frac{\partial^3}{\partial x_l\partial x_3^2}\left(\frac{e^{-\lambda_3 r}-1}{r}-\frac{\lambda_3^2}{2}r\right)+\frac{\lambda+\mu}{\lambda+2\mu}x_3\frac{\partial^2}{\partial x_l\partial x_3}\frac{1}{r}\right]+\\ &+\delta_{l3}(1-\delta_{3j})\left[\alpha_{12}\frac{\partial^2}{\partial x_j\partial x_3}\frac{e^{-\lambda_3 r}-1}{r}+\frac{\alpha_2}{\mu(\alpha_2+1)}x_3\frac{\partial}{\partial x_j}\frac{1}{r}\right]+\\ &+\delta_{l3}\delta_{3j}\left[-\frac{\partial}{\partial x_3}\frac{1}{r}+\frac{\lambda+\mu}{\lambda+2\mu}x_3\frac{\partial^2}{\partial x_3^2}\frac{1}{r}+\alpha_{14}\frac{\partial^3}{\partial x_3^3}\left(\frac{e^{-\lambda_3 r}-1}{r}-\frac{\lambda_3^2}{2}r\right)-\right.\\ &\left.-\alpha_{14}\lambda_3^2\frac{\partial}{\partial x_3}\frac{e^{-\lambda_3 r}-1}{r}\right]\end{aligned}$$

$$\mathbf{K}_{lj}^{(2)}(x,y) = (1-\delta_{l3})(1-\delta_{3j})\frac{\partial^2}{\partial x_l \partial x_j}(\alpha_{13}\lambda_7^2 r - \Psi_1(r)) +$$
$$+\delta_{3j}(1-\delta_{l3})\left(-\frac{\partial^3\Psi_2(r)}{\partial x_l \partial x_3^2} + \frac{\partial\Psi_3(r)}{\partial x_l} - \alpha_{17}\left(x_3\frac{\partial}{\partial x_3} - 1\right)\frac{\partial}{\partial x_l}\frac{1}{r}\right) -$$
$$-\delta_{l3}(1-\delta_{3j})\left(\frac{\partial^2\Psi_1(r)}{\partial x_l \partial x_3} + \frac{\alpha_3\varkappa_1}{\alpha_6\varkappa_2(\alpha_2+1)}x_3\frac{\partial}{\partial x_j}\frac{1}{r}\right) +$$
$$+\delta_{l3}\delta_{3j}\left(\frac{\partial\Psi_3(r)}{\partial x_3} - \frac{\partial^3\Psi_2(r)}{\partial x_3^3} - \alpha_{17}x_3\frac{\partial^2}{\partial x_3^2}\frac{1}{r}\right),$$

$$\mathbf{K}_{lj}^{(4)}(x,y) = (1-\delta_{l3})(1-\delta_{3j})\left(-\frac{1}{\varkappa_6}\delta_{lj}\frac{e^{-\lambda_2 r}}{r} + \frac{\partial^2\Psi_5(r)}{\partial x_l \partial x_j}\right) +$$
$$+\delta_{3j}(1-\delta_{l3})\frac{\partial}{\partial x_l}\left(\frac{\lambda_1^2-\lambda_4^2}{\lambda_4^2}\frac{e^{-\lambda_1 r}}{r} + \frac{\partial^2}{\partial x_3^2}(\Psi_7(r) + \alpha_{18}r)\right) +$$
$$+\delta_{l3}(1-\delta_{3j})\frac{\partial^2\Psi_5(r)}{\partial x_3 \partial x_j} + \delta_{l3}\delta_{3j}\left(-\frac{\partial}{\partial x_3}\frac{1}{r} + \frac{\partial^3\Psi_7(r)}{\partial x_3^3} + \frac{1}{\lambda_4^2}\frac{\partial\Psi_6(r)}{\partial x_3} +\right.$$
$$\left.+\alpha_{18}x_3\frac{\partial}{\partial x_3}\frac{1}{r}\right),$$

$$\mathbf{K}_{l1}^{5}(x,y) = (1-\delta_{l3})\left(\frac{\partial\Psi_4(r)}{\partial x_l} + \frac{\alpha_3}{\alpha_6(\alpha_2+1)}\left(x_3\frac{\partial}{\partial x_3} - 1\right)\frac{\partial\Phi(x,y)}{\partial x_l}\right) +$$
$$+\delta_{l3}\left(\frac{\alpha_3}{\alpha_6(\alpha_2+1)}\frac{x_3}{r} + \frac{\partial\Psi_4(r)}{\partial x_3}\right),$$

$$\mathbf{K}_{l1}^{(6)}(x,y) = \frac{\eta}{\eta_1(\lambda+2\mu)\lambda_3^2}\frac{\partial}{\partial x_l}\frac{e^{-\lambda_3 r}-1}{r},$$

$$\mathbf{K}_{l1}^{(7)}(x,y) = \frac{\alpha_4}{\alpha_6}\frac{\partial}{\partial x_l}\frac{e^{-\lambda_1 r}-1}{r},$$

$$\mathbf{K}_{1j}^{(10)}(x,y) = (1-\delta_{3j})\frac{\varkappa_1}{\alpha_6\varkappa_2}\frac{\partial}{\partial x_j}\frac{e^{-\lambda_1 r}-1}{r} +$$
$$+\delta_{3j}\frac{\varkappa_1}{\varkappa\lambda_4^2}\left(\frac{\partial^2}{\partial x_3^2}\frac{e^{-\lambda_1 r}-1}{r} - (\lambda_1^2-\lambda_4^2)\frac{e^{-\lambda_1 r}}{r}\right),$$

$$\mathbf{K}_{1j}^{(11)}(x,y) = -\frac{\alpha_5}{\mu(\alpha_2+1)}(1-\delta_{3j})\frac{\partial}{\partial x_j}\frac{e^{-\lambda_3 r}-1}{r} +$$
$$+\delta_{3j}\frac{2\alpha_5}{\alpha_2+1}\left(\lambda_3^2\frac{e^{-\lambda_3 r}}{r} - \frac{\partial^2}{\partial x_3^2}\frac{e^{-\lambda_3 r}-1}{r}\right),$$

$$\mathbf{K}_{1j}^{(12)}(x,y) = (1-\delta_{3j})\frac{\partial}{\partial x_j}\left(\frac{\varkappa_1\alpha_3\alpha_5}{\alpha_6\varkappa_2(\alpha_2+1)}\frac{1}{r} - \frac{\alpha_2\varkappa}{\alpha_5\alpha_6\varkappa_2}\frac{e^{-\lambda_1 r}-1}{r} +\right.$$
$$\left.+\alpha_9\frac{e^{-\lambda_3 r}-1}{r}\right) + \delta_{3j}\left(\Psi_8(r) + \frac{\partial^2}{\partial x_3^2}\Psi_9(r)\right),$$

$$\mathbf{K}^{(13)}(x,y) = -\frac{1}{\alpha_6}\left(\frac{\alpha_4\varkappa_1}{\varkappa}\frac{e^{-\lambda_1 r}-1}{r} + \frac{1}{r}\right),$$

$$\mathbf{K}^{(15)}(x,y) = \alpha_{10}\frac{e^{-\lambda_3 r}-1}{r} + \frac{\alpha_4\alpha_2}{\alpha_5\alpha_6}\frac{e^{-\lambda_1 r}-1}{r},$$

$$\mathbf{K}^{(16)}(x,y) = -\frac{1}{\eta}\frac{e^{-\lambda_3 r}}{r},$$

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$$
\begin{aligned}
\Psi_1(r) &= \frac{\varkappa a_3}{\alpha_5\alpha_6\varkappa_2}\frac{e^{-\lambda_1 r}-1}{r} + \frac{\eta\alpha_9}{(\lambda+2\mu)\lambda_3^2}\frac{e^{-\lambda_3 r}-1}{r},\\
\Psi_2(r) &= \frac{a_3}{\alpha_5\lambda_4^2}\left(\frac{e^{-\lambda_1 r}-1}{r} - \frac{1}{2}\lambda_1^2 r\right) + \frac{\eta\alpha_{11}}{(\lambda+2\mu)\lambda_3^2}\left(\frac{e^{-\lambda_3 r}-1}{r} - \frac{1}{2}\lambda_3^2 r\right)\\
\Psi_3(r) &= \frac{a_3(\lambda_1^2-\lambda_4^2)}{\alpha_5\lambda_4^2}\frac{e^{-\lambda_1 r}-1}{r} + \frac{\eta\alpha_{11}(\lambda_3^2-\lambda_5^2)}{(\lambda+2\mu)\lambda_3^2}\frac{e^{-\lambda_3 r}-1}{r},\\
\alpha_{17}(r) &= \frac{a_3\lambda_1^2}{2\alpha_5\lambda_4^2} + \frac{\alpha_{11}\eta}{2(\lambda+2\mu)} - \frac{\varkappa_1 a_3}{\varkappa(\alpha_2+1)\lambda_4^2},\\
\Psi_4(r) &= \frac{a_3\alpha_4}{\alpha_5\alpha_6}\frac{e^{-\lambda_1 r}-1}{r} - \frac{\eta\alpha_{10}}{(\lambda+2\mu)\lambda_3^2}\frac{e^{-\lambda_3 r}-1}{r},\\
\Psi_5(r) &= \frac{1}{\varkappa_2}\frac{e^{-\lambda_2 r}-1}{r} - \frac{\varkappa}{\varkappa_2\alpha_6}\frac{e^{-\lambda_1 r}-1}{r},\\
\Psi_6(r) &= \frac{\alpha_6\lambda_2^2}{\varkappa\lambda_4^2}\frac{e^{-\lambda_2 r}-1}{r} + \frac{\lambda_1^2-\lambda_4^2}{\lambda_4^2}\frac{e^{-\lambda_1 r}-1}{r},\\
\Psi_7(r) &= \frac{\alpha_6}{\varkappa\lambda_4^2}\left(\frac{e^{-\lambda_2 r}-1}{r} - \frac{1}{2}\lambda_2^2 r\right) - \frac{1}{\lambda_4^2}\left(\frac{e^{-\lambda_1 r}-1}{r} - \frac{1}{2}\lambda_1^2 r\right),\\
\alpha_{18} &= \frac{\lambda_2^2\alpha_6 - \varkappa\lambda_1^2}{2\varkappa\lambda_4^2},\\
\Psi_8(r) &= \frac{a_3(\lambda_1^2-\lambda_4^2)}{\alpha_5\lambda_4^2}\frac{e^{-\lambda_1 r}}{r} - \alpha_{11}(\lambda_3^2-\lambda_5^2)\frac{e^{-\lambda_3 r}}{r},\\
\Psi_9(r) &= \alpha_{11}\frac{e^{-\lambda_3 r}-1}{r} - \frac{a_2}{\alpha_5\lambda_4^2}\frac{e^{-\lambda_1 r}-1}{r}.
\end{aligned}
$$

Let us introduce the generalized thermostress operators

$$
\mathbf{P}(\partial,n)U = \begin{bmatrix}
\mathbf{T}^{(1)}(\partial,n) & [0]_{3\times3} & -\gamma n^\top & \eta n^\top\\
[0]_{3\times3} & \mathbf{T}^{(2)}(\partial,n) & [0]_{3\times1} & [0]_{3\times1}\\
[0]_{1\times3} & \varkappa_1 n & \varkappa\dfrac{\partial}{\partial n} & 0\\
[0]_{1\times3} & -\varkappa_7 n & o & \eta_1\dfrac{\partial}{\partial n}
\end{bmatrix}_{8\times8}
$$

where $n=(0,0,1)$ is a normal vector

$$
\mathbf{T}^{(p)}(\partial,n) = \left[\mathbf{T}_{kl}^{(p)}(\partial,n)\right]_{3\times3}, \quad p=1,2,
$$

$$
\mathbf{T}_{kl}^{(1)}(\partial,n) = \mu\delta_{kl}\frac{\partial}{\partial x_3} + \lambda\delta_{k3}\frac{\partial}{\partial x_l} + \mu\delta_{3l}\frac{\partial}{\partial x_k}, \quad k,l=1,2,3,
$$

$$
\mathbf{T}_{kl}^{(2)}(\partial,n) = \varkappa_6\delta_{kl}\frac{\partial}{\partial x_3} + \varkappa_4\delta_{k3}\frac{\partial}{\partial x_l} + \varkappa_5\delta_{3l}\frac{\partial}{\partial x_k}, \quad k,l=1,2,3.
$$

Calculate the stress vector $\mathbf{P}(\partial,n)U$ by (4.9), we have

$$
\mathbf{P}(\partial,n)U(x) = \frac{1}{2\pi}\iint_{-\infty}^{+\infty} L(x,y)f(y)\,dy_1dy_2, \tag{4.10}
$$

where

$$
\mathbf{L}(x,y) = \begin{bmatrix}
\mathbf{L}^{(1)}(x,y) & \mathbf{L}^{(2)}(x,y) & \mathbf{L}^{(5)}(x,y) & \mathbf{L}^{(6)}(x,y)\\
[0]_{3\times3} & \mathbf{L}^{(4)}(x,y) & \mathbf{L}^{(7)}(x,y) & [0]_{3\times1}\\
[0]_{1\times3} & \mathbf{L}^{(10)}(x,y) & \mathbf{L}^{(13)}(x,y) & 0\\
\mathbf{L}^{(11)}(x,y) & \mathbf{L}^{(12)}(x,y) & \mathbf{L}^{(15)}(x,y) & \mathbf{L}^{(16)}(x,y)
\end{bmatrix}_{8\times8},
$$

$$\mathbf{L}^{(p)}(x,y) = \left[\mathbf{L}_{kj}^{(p)}(x,y)\right]_{3\times3}, \quad p=1,2,3,4, \quad \mathbf{L}^{(p)}(x,y) = \left[\mathbf{L}_{kj}^{(p)}(x,y)\right]_{3\times1}, \quad p=5,6,7,8,$$
$$\mathbf{L}^{(p)}(x,y) = \left[\mathbf{L}_{kj}^{(p)}(x,y)\right]_{1\times3}, \quad p=9,10,11,12, \quad \mathbf{L}^{(p)}(x,y), \quad p=13,14,15,16,$$

are scalar functions,

$$\mathbf{L}_{kj}^{(p)}(x,y) = \sum_{l=1}^{3} \mathbf{T}_{kl}^{(1)}(\partial,n)\mathbf{K}_{lj}^{(p)}(x,y) + (\eta\mathbf{K}_{1j}^{(10+p)}(x,y) - \gamma\delta_{2p}\mathbf{K}_{lj}^{(10)}(x,y))\delta_{k3}, \quad p=1,2,$$
$$\mathbf{L}_{kj}^{(4)}(x,y) = \sum_{l=1}^{3} \mathbf{T}_{kl}^{(2)}(\partial,n)\mathbf{K}_{lj}^{(4)}(x,y),$$
$$\mathbf{L}_{k1}^{(p)}(x,y) = \sum_{l=1}^{3} \mathbf{T}_{kl}^{(1)}(\partial,n)\mathbf{K}_{l1}^{(p)}(x,y) + (\eta\mathbf{K}^{(10+p)}(x,y) - \gamma\delta_{p5}\mathbf{K}^{(13)}(x,y))\delta_{k3}, \quad p=5,6,$$
$$\mathbf{L}_{k1}^{(7)}(x,y) = \sum_{l=1}^{3} \mathbf{T}_{kl}^{(2)}(\partial,n)\mathbf{K}_{l1}^{(7)}(x,y),$$
$$\mathbf{L}_{1j}^{(10)}(x,y) = \varkappa\frac{\partial}{\partial x_3}\mathbf{K}_{1j}^{(10)}(x,y) + \varkappa_1\mathbf{K}_{1j}^{(4)}(x,y),$$
$$\mathbf{L}_{1j}^{(11)}(x,y) = \eta_1\frac{\partial}{\partial x_3}\mathbf{K}_{1j}^{(11)}(x,y), \quad \mathbf{L}_{1j}^{(12)}(x,y) = \eta\frac{\partial}{\partial x_3}\mathbf{K}_{1j}^{(12)}(x,y) - \varkappa_7\mathbf{K}_{1j}^{(4)}(x,y),$$
$$\mathbf{L}^{(13)}(x,y) = \varkappa\frac{\partial}{\partial x_3}\mathbf{K}^{(13)}(x,y) + \varkappa_1\mathbf{K}_{11}^{(7)}(x,y),$$
$$\mathbf{L}^{(15)}(x,y) = \eta\frac{\partial}{\partial x_3}\mathbf{K}_{1j}^{(15)}(x,y) - \varkappa_7\mathbf{K}_{11}^{(7)}(x,y),$$
$$\mathbf{L}^{(16)}(x,y) = \eta_1\frac{\partial}{\partial x_3}\mathbf{K}^{(16)}(x,y).$$

From the equations (4.6), we obtained

$$\hat{f}_j{}'(0)=0, \quad \hat{f}_j{}''(0)=0, \quad j=1,2, \quad \hat{f}_4{}'(0)=0.$$

If in this equality we use (4.7), then we obtain

$$\iint_{-\infty}^{+\infty} f'_j(y)dy_1dy_2 = 0, \quad \iint_{-\infty}^{+\infty} f''_j(y)dy_1dy_2 = 0, \quad j=1,2$$
$$\iint_{-\infty}^{+\infty} f'_4(y)dy_1dy_2 = 0. \tag{4.11}$$

Thus (4.11) provides the convergence of integrals (4.9) and (4.10) if the vector $f(y)$ is smooth. Assume that the functions $f'_3(y),\ f''_3(y) \in C^{1,\alpha}(\partial\Omega),\ f'_4(y),\ f''_4(y),\ f'_j(y),\ f''_j(y) \in C^{0,\alpha}(\partial\Omega),\ j=1,2,\ 0<\alpha<1$, then by straight forward verification we establish that the vector $U(x)$ represented in form (4.9) is a solution of system (2.1)-(2.4) in the domain Ω^-. If in the functions $u_3(x)$ and $w_3(x)$ from (4.9), the functions $\{\mathbf{P}^{(1)}(\partial,n)U'(x)\}_j$, $\{\mathbf{P}^{(2)}(\partial,n)w(x)\}_j, j=1,2, \mathbf{P}^{(3)}(\partial,n)U''(x), \mathbf{P}^{(4)}(\partial,n)U'''(x)$ from (4.10) we pass to the limit as $x\to y\in\partial\Omega$ $(x_3\to 0)$ and take into account that the kernels of the integral representation of these functions are equal to zero when and also $x_3=0$,

$$\lim_{x\to y}\frac{1}{2\pi}\iint_{-\infty}^{+\infty}\frac{\partial}{\partial x_3}\frac{1}{r}f(y)\ dy_1dy_2 = -f(y), \quad y\in\partial\Omega,$$

We obtain that the vector $U(x)$ represented in form (4.9) satisfies the boundary conditions (3.1).

If the boundary vector-function, satisfies the conditions

$$|f'_3(y)|,\ \ |f''_3(y)| < \frac{A}{1+|y|},\ \ |f'_4(y)|,\ \ |f''_4(y)|,\ \ |f'_j(y)|,\ \ |f''_j(y)| < \frac{A}{1+|y|^2},$$
$$j = 1,2, \quad y \in \partial\Omega, \quad A = const > o,$$

then the vector $U(x)$ represented by formula (4.9) is a regular solution of problem $(III)^-$ which satisfies the following decay conditions at infinity

$$u_3(x),\ \ w_3(x) = O(|x|^{-1}),\ \ \partial_k u_3(x),\ \ \partial_k w_3(x) = O(|x|^{-2}\ln|x|),\ \ k = 1,2,3,$$
$$u_j(x),\ \ w_j(x),\ \ \theta(x),\ \ v(x) = O(|x|^{-1}\ln|x|),\ \ j = 1,2,$$
$$\partial_k u_j(x),\ \ \partial_k w_j(x),\ \ \partial_k \theta(x),\ \ \partial_k v(x) = O(|x|^{-2}),\ \ k = 1,2,3,\ \ j = 1,2,\ \ \partial_k = \frac{\partial}{\partial x}$$

Conclusion. Analogousy can be treated the IV type boundary value problem for a half-space.

References

1. J. Barber, The solution of elasticity problems for the half-space by the method of Green and Collins, Applied Scientific Research, 40 (1983),135-157.
2. M. Basheleishvili, L. Bitsadze., Explicit solutions of the boundary value problems of the theory of consolidation with double porosity for half-plane, Georgian Mathematical Journal, 19 (2012), 41-48.
3. M. Basheleishvili, L. Bitsadze., Explicit solutions of the boundary value problems of the theory of consolidation with double porosity for half-plane, Buletin of TICMI, 14 (2010), 9-15.
4. D.Burchuladze, M. Kharashvili, K. Skhvitaridze., An effective solution of the Dirichlet problem for a half-space with double porosity, Georgian int. Journal of science and tehnology, Nova Science Publishers, 3 (2011),vol.3 223-232.
5. P.S. Casas, R. Quintanilla, Exponential stability in thermoelasticity with microtemperatures, Internat. J. Eng.Sci., 43 (2005),33-47.
6. L. Giorgashvili, K. Skhvitaridze, M. Kharashvili., Effective solution of the Neumann boundary value problem for a half-space with double porosity, Georgian int. Journal of science and technology, Nova Science Publishers, Inc., (2012),143 - 154.
7. L.Giorgashvili, Sh.ZazaSvili, R.Meladze., AbouT on explicit solution method for boundary value problems of elastostatics for half-space. Nova Science Publishers, Inc. Vol. 6, 3 (2014),215 - 224.
8. L. Giorgashvili, D. Metreveli., Problems of Statics of Two-Component Elastic mixtures for Half-Space. Proc. A.Razmadze Math. Inst. Vol.167, (2015),43-61.
9. D.Iesan., Thermoelasticity of bodies with microstructure and microtemperatures, Int.J. Solids and Structures, 44 (2007), 8648–8662.
10. R. Kumar, T. Chadha., Plane problem in micropolar thermoplastic half-space with stretch, Indian J. pure appl. Math., 176 (1986),827 - 842.
11. V. Kupradze, T. Gegelia, M. Basheleishvili and T. Burchuladze, Three-dimensional problems of the mathematical theory of elasticity and thermoelectricity, Nauka, Moscow, 1976.
12. H. Sherief, H. Saleh., A half-space problem in the theory of generalized thermoplastic diffusion. Int. J. of Solids and Structures, 42 (2005),4484-4493.
13. B. Sigh, R. Kumar., Reflection of plane waves from the flat boundary of a micropolar generalized thermoplastic half-space, Int. J. of Engineering Science, 36 (1998),866-890.
14. K.Skhvitaridze, M.Kharashvili., Investigation of the Dirichlet and Neumann Boundary value problems for a hslf-space filled a viscous incompressible fluid., Mechanics o the continuous environment issues., Published by Nova science Publishers, Inc. New Yorc, (2012),85-98.
15. M. Svanadze., On the linear of thermoelectricity with microtemperatures, Tec. Mechanik, 32, 2-5 (2012), 564–576.

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ანოტაცია. იზოტროპიული მიკროსტრუქტურის მქონე სხეულებისთვის თერმოდინამიკის დიფერენციალურ განტოლებათა სისტემა გადაადგილების ვექტორის, მიკრობრუნვის ვექტორის, მიკროტემპერატურის ვექტორის, მიკროდაჭიმულობისა და ტემპერატურული ცვლილების ფუნქციების მიმართ წარმოდგენილია კომპლექსური მეორე გვარის კერძოწარმოებულიანი დიფერენციალურ განტოლებათა სისტემით.

მოცემულია თერმოდრეკადობის წრფივი თეორიის სტატიკის შემთხვევა მიკროდაჭიმულობისა და მიკროტემპერატურის გათვალისწინებით. მიღებულია დიფერენციალურ განტოლებათა სისტემის ზოგადი ამოხსნის ფორმულა, გამოსახული ოთხი ჰარმონიული და ოთხი მეტაჰარმონიული ფუნქციებით. მიღებული ფორმულა მეტად მოხერხებული და მნიშვნელოვანია მრავალი კერძო ამოცანის ამოსახსნელად კონკრეტული გეომეტრიის მქონე სხეულებისთვის; განხილულია III ტიპის ამოცანა ნახევარსივრცისათვის და დამტკიცებულია ამონახსნის ერთადერთობის თეორემა (ამონახსნები მიღებულია კვადრატურებში).

საკვანძო სიტყვები: თერმოდრეკადობა; მიკროდაჭიმულობა; მიკროტემპერატურა; ფუნქცია; ფურიეს გარდაქმნა.

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Задача статики линейной теории термоупругости для тел с микрорастяжением с учетом микротемпературы для полупространства

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Аннотация. Система дифференциальных уравнений для изотропного микроструктурного упругого тела, выраженной через векторное перемещание, вектор микротемпературы, вектор микровращений, функции микрорастяжений и температуры, представлена системой комплексных уравнений с частными производными второго порядка.

В статье рассмотрен случай статики линейной теории термоупругости с учетом микрорастяжений и микротемпературы. Получена формула представлений общих решений системы дифференциальных уравнений с помощью четырех гармонических и метагармонических функций. Полученная формула является более удобней при решении частных задач для тел конкретной геометрии. В работе рассмотрена задача III –го типа для полупространства. Даказана теорема единственности этой задачи. Решение получено в квадратурах.

Ключевые слова: микротемпература; микрорастяжение; преобразование Фурье; термоупругость; функция.

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