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Inequality of Plasticity, Bending with Stretching

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Abstract.

for computation of the rotating disk with a curved rod was selected such a calculation scheme which is taking into account complex deformation, i. e. along with the action of the bending moment, the action of stretching force was also taken into account. The elastic and plastic analysis of this phenomenon is given. The angular rotation speeds of the asymmetric rod are calculated and their critical values were found. The effect of the disk size, on the one hand and the size of the rod, on the other hand, is shown on the stress-strain state of the mechanism. The limited straight line of the border of change of the values of the bending moment and the stretching force was obtained, at which plastic zones in the system under consideration appear and expand. The analytical expressions of the normal stresses arisen at such loads is given.

Keywords: Angular speed of rotation; Complex deformation; Elastic analysis; Fluidity limit; Plastic analysis; Radial force; Stresses of stretching and bending.

Introduction

The study and solution of the problem presented in this work is due to the following circumstances. In the aviation and shipping industries, also in different designs and details of the machines, the primary parts of the structure (e. g. propellers and the like) experience complex deformation - bending with stretching during operation at reality. When calculating such structures as we managed to find out from the relevant literature, only bending deformation is accepted in mind, which does not correspond to the real picture. The purpose of this work is to find critical angular rotation speed of the asymmetric rod. On fig. 1 a cylindrical disk and flat radial rod are shown in the profile and from above. Under the influence of centrifugal force, the rod will try to straighten up, therefore, on each section will act both the bending moment and the force of stretching (which has not been taken into account yet). Due to the action of gravity, a small transverse force also arises, but it can be neglected. To simplify the calculation, consider the rod with a flat base.

The solution to the problem requires the consideration of both elastic and plastic analysis..

Main Part

Elastic analysis

The full tensile force or radial force acting on the cross section of height h_2 will be

$$N = \int_r^b \rho \cdot htS\omega^2 ds = \frac{\rho \cdot t\omega^2 h_0}{6L}[b^3 - r^2(3b - 2r)] \quad (1)$$

If you consider that $h/_{h_0} = (b - s)\lambda$ is the average stress of the tension along the cross section

$$\sigma_t = \frac{N}{h_2 t} = \frac{\rho \cdot \omega^2}{6(b - r)}[b^3 - r^2(3b - 2r)] = \frac{\rho \cdot \omega^2}{6}[b(b + r) - 2r^2] \quad (2)$$

A full bending moment, also acting in the cross - section regarding its centre is

$$M = \int_r^b \rho htS\omega^2 \left(\frac{h_2 - h}{h}\right) dS \quad (3)$$

The maximum bending stress occurs in the most extreme fibers of the rod, and therefore

$$\sigma_m^{max} = (b^2 - r^2)\rho \cdot \omega^2/_4 \quad (4)$$

The most tensile stress in the cross section h_2 is created in the lowest layers. Connecting equations (2) and (4) we get the fluidity stress

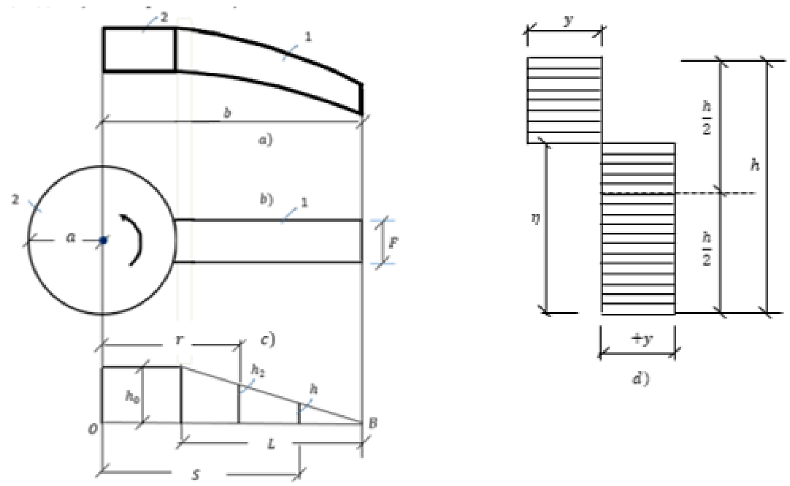

Fig. 1

$$\sigma_m = \frac{\rho \cdot \omega^2}{12}[3(b^2 - r^2) + 2b(b + r) - 4r^2] = \frac{\rho \cdot \omega^2}{12}(5b^2 + 2br - 7r^2) \quad (5)$$

In the rods with diameter of the disk $< b/7$, the largest stress occurs at a distance $r = b/_7$ and $\sigma_m = \frac{3\rho\omega^2 b^2}{7}$. If the diameter of the disk $> b/7$, then σ_m arises where the rod is connected to the disk and is equally to $\frac{\rho\omega^2(b-r)(5b+7r)}{12}$. Therefore, at completely elastic loading and the condition that $\sigma_m = y$, i. e. is equal to the yield limit for stretching

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$$\left.\begin{aligned} a)\ {}^{b}/_{7} > 2r;\ \omega^2 \le \frac{7\cdot y}{3\rho b^2}; \\ b)\ {}^{b}/_{7} < 2r;\ \omega^2 \le \frac{12y}{\rho(b-2r)(5b+7\cdot 2r)} \end{aligned}\right\} \quad (6)$$

Plastic analysis

When the rotational speed exceeds the speed given by equation (6), plastic deformations appear in the lower part of the rod cross-section. Further increase causes an increase of the plastic region and finally plastic deformations occur in the upper part of the rod cross-section as well. We believe that the material is perfectly plastic. The reached fluidity stress will no longer increase. Two plasticity zones are separated by an elastic zone. We're not going to think about the growth of these zones with increasing ω, which is embraced completely by plastic deformations earlier than others. The speed at which this will happen is the limit for the rod, since it becomes possible immediately a large deflection.

For simultaneous tensile and bending, the stress distribution in the cross-section fully embraced by plastic deformation is shown in (Fig. 1, d), where η determines the location of the stresses. The resultant of tensile stresses acting in the cross-section is $N = 2ty\left(\eta - \frac{h}{2}\right)$ and the largest moment $M = ty\eta(h-\eta)$ at $\eta = h$. The stresses distribution is the same as in the case of pure tension, when the tensile force is $N_0 = t\cdot y\cdot h$; and at $\eta = \frac{h}{2}$, as in the case of pure bending, bending moment $M_0 = \frac{tyh^2}{4}$. Thus

$$\left(\frac{N}{N_0}\right)^2 = \left(\frac{\eta-\frac{h}{2}}{\frac{h}{2}}\right)^2 \text{ and } \frac{M}{M_0} = \frac{\eta(h-\eta)}{\frac{h^2}{4}} \quad (7)$$

Excluding η from these equations, we obtain

$$\left(\frac{N}{N_0}\right)^2 + \frac{M}{M_0} = 1$$

It is obvious that after plastic deformation of the whole cross-section there should be no stress greater than σ_y. We can write down the plasticity inequality

$$\left(\frac{N}{N_0}\right)^2 + \frac{M}{M_0} = 1 \quad (8)$$

In the first inequality we substitute M from (1) and (3), then we get

$$\left.\begin{aligned} \left(\frac{N}{N_0}\right)^2 = \left(\frac{\rho\cdot\omega^2}{6y}\right)[b(b+r)-2r^2] \\ \frac{M}{M_0} = \left(\frac{\rho\cdot\omega^2}{6y}\right)(b^2-r^2) \end{aligned}\right\} \quad (9)$$

Assuming $\frac{\rho\cdot\omega^2}{6y} = P$ and writing x instead of $\left(1-\frac{r}{b}\right)$, we get

$$P^2\cdot x^2(3-2x)^2 + Px(2-x) - 1 \le 0$$

Setting values from x to 0,6 we calculate the corresponding positive values of P

x	1	0,9	0,8	0,6
P	0,615	0.594	0.592	0,63

Obviously, P takes the minimum value $P \approx 0{,}59$, at $x \approx 0{,}82$. This means that the inequality is true for every cross-section defined by x, if P, and therefore ω too, are less of the computed value. The maximum permissible angular speed of the rod corresponds to the least from the maximum values P, hence $0{,}59 = \frac{\rho\omega^2 b^2}{6y}$ or $\omega = 1{,}88\sqrt{\frac{y}{\rho b^2}}$. This solution can be used in that case, if $a < 0{,}18b$ or $6{,}5\cdot 2r < b$. Thus using (6), we get

$a)\ 2r < 0{,}18b \qquad \omega_P = 1{,}88\sqrt{\frac{y}{\rho b^2}}$

$b)\ 2r < \frac{b}{7} \qquad \omega_E = 1{,}53\sqrt{\frac{y}{\rho b^2}}$

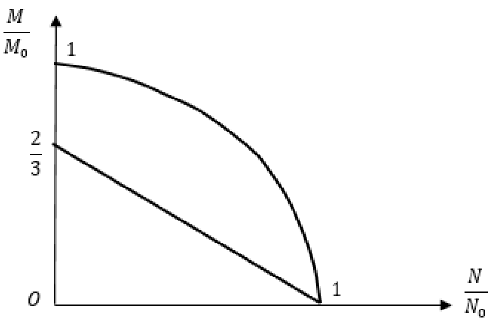

Fig. 2

where, ω_E angular speed, at which the first signs of fluidity appear. The meaning of the results is visible from (Fig. 2). We built a) inequality of plasticity b)

relationship between N and M, which is such that when they act together, then they cause fluidity stresses. Suppose σ – homogeneous stress of stretching in the cross-section; then $\frac{N}{N_0} = \frac{\sigma}{y}$ and thus

$$\frac{M}{M_0} = \frac{bh^2(y-b)}{6} \cdot \frac{4}{bh^2 y} = \frac{2}{3}\left(1 - \frac{\sigma}{y}\right) = \frac{2}{3}\left(1 - \frac{N}{N_0}\right)$$

This expression describes the straight line on the fig. 2. All possible combinations M and N providing the appearance of plastic deformations are determined by the points located between these two lines. Note that the small value N almost does not influence into the amount of the moment necessary to achieve the limit of fluidity.

Conclusion

The presented work provides evidence that when solving the tasks of the considered class, it cannot be neglected by expansion deformation. The formulas and dependences between N and M are bred that clearly demonstrating this assumption. Shown at what ratio N and M increases the effect of stretching on the overall picture of deformation. The role of tensile strain in achieving plastic deformation and plastic flow is pointed out.

It is shown how the ratio of the disk dimensions, on the one hand, and the rod dimensions, on the other hand, affects the stress-strain state of the mechanism. Both elastic and plastic analysis of this phenomenon is given.

References

1. Brown, E. H. (1967). Plastic asymmetrical bending of beams. *International Journal of Mechanics*.
2. Prager, W. (n.d.). Limit design of beams and frames. *Proceedings of the American Society of Civil Engineers*.
3. Nadai, A. (n.d.). *Theory of flow and fracture of solids* (Vol. 1). McGraw-Hill.
4. Batsikadze, T., Kvaratskhelia, A., & Mazaghua, Z. (2014). *A brief course in the theories of elasticity, plasticity, and creep* (in Georgian). Publishing House "Technical University", Tbilisi.
5. Batsikadze, T., Nizharadze, J., & Giorgobiani, R. (2024). Analysis of plastic torsion using physical analogies. *Scientific-Technical Journal "Building", 2*(70), Tbilisi. *(in Georgian with English abstract)*

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